JEE (Main) MATHEMATICS SOLVED PAPER

2021
20th July Shift 1

Time : 1 Hour **Total Marks : 100**

General Instructions :

1. *In Chemistry Section, there are 30 Questions (Q. no. 1 to 30).*
2. *In Chemistry, Section A consists of 20 multiple choice questions & Section B consists of 10 numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
3. *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
5. *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
6. *All calculations / written work should be done in the rough sheet is provided with Question Paper.*

Mathematics

Section A

Q. 1. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is:

(1) $6+8\sqrt{3}$ **(2)** $8+2\sqrt{2}$

(3) $4+2\sqrt{3}$ **(4)** $10+6\sqrt{2}$

Q. 2. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is:

(1) $\frac{1}{9}$ **(2)** $\frac{1}{66}$

(3) $\frac{2}{11}$ **(4)** $\frac{1}{11}$

Q. 3. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are:

(1) 10, 11 **(2)** 8, 13

(3) 1, 20 **(4)** 3, 18

Q. 4. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$, angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|(\vec{a}\times\vec{b})\times\vec{c}|$ is:

(1) $\frac{2}{3}$ **(2)** 4

(3) 3 **(4)** $\frac{3}{2}$

Q. 5. The value of the integral $\int_{-1}^{1} \log_e\left(\sqrt{1-x} + \sqrt{1+x}\right) dx$ is equal to:

(1) $2\log_e 2 + \frac{\pi}{4} - 1$ **(2)** $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$

(3) $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$ **(4)** $\log_e 2 + \frac{\pi}{2} - 1$

Q. 6. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in$ R, is :

(1) $\frac{1}{4}$ **(2)** $\frac{7}{36}$

(3) $\frac{2}{9}$ **(4)** $\frac{1}{6}$

Q. 7. Let $y = y(x)$ be the solution of the differential equation

$$x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx,\ -1 \le x \le 1,$$

$y\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

Then the area of the region bounded by the curves $x = 0$, $x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is :

(1) $\frac{1}{12}(\pi - 3)$ **(2)** $\frac{1}{6}(\pi - 1)$

(3) $\frac{1}{8}(\pi - 1)$ **(4)** $\frac{1}{4}(\pi - 2)$

Q. 8. If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to:

(1) 56×3^{25} **(2)** 52×3^{24}

(3) 56×3^{24} **(4)** 28×3^{25}

Q. 9. Let a function $f: R \to R$ be defined as

$$f(x) = \begin{cases} \sin x - e^x, & \text{if } x \le 0 \\ a + [-x], & \text{if } 0 < x < 1 \\ 2x - b, & \text{if } x \ge 1 \end{cases}$$

where $[x]$ is the greatest integer less than or equal to x. If f is continuous on R, then $(a + b)$ is equal to:

(1) 5 **(2)** 3

(3) 2 **(4)** 4

Q. 10. Let $y = y(x)$ be the solution of the differential equation $e^x\sqrt{1 - y^2}\,dx + \left(\frac{y}{x}\right)dy = 0$, $y(1) = -1$. Then, the value of $(y(3))^2$ is equal to:

(1) $1 + 4e^3$ **(2)** $1 + 4e^6$

(3) $1 - 4e^6$ **(4)** $1 - 4e^3$

Q. 11. If z and ω are two complex numbers such that $|z\omega| = 1$ and $arg(z) - arg(\omega) = \frac{3\pi}{2}$, then $arg\left(\frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega}\right)$ is:

(Here $arg(z)$ denotes the principal argument of complex number z)

(1) $\frac{3\pi}{4}$ **(2)** $-\frac{\pi}{4}$

(3) $-\frac{3\pi}{4}$ **(4)** $\frac{\pi}{4}$

Q. 12. Let $[x]$ denote the greatest integer $\le x$, where $x \in R$. If the domain of the real valued function

$f(x) = \sqrt{\frac{|[x]| - 2}{|[x]| - 3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty)$, $a < b < c$, then the value of $a + b + c$ is:

(1) –3 **(2)** 1

(3) –2 **(4)** 8

Q. 13. The number of real roots of the equation

$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{4}$ is:

(1) 0 **(2)** 4

(3) 1 **(4)** 2

Q. 14. The coefficient of x^{256} in the expansion of $(1 - x)^{101}(x^2 + x + 1)^{100}$ is:

(1) $-{}^{100}C_{16}$ **(2)** ${}^{100}C_{16}$

(3) ${}^{100}C_{15}$ **(4)** $-{}^{100}C_{15}$

Q. 15. Let the tangent to the parabola $S : y^2 = 2x$ at the point P(2, 2) meet the x–axis at Q and normal at it meet the parabola S at the point R. Then, the area (in sq. units) of the triangle PQR is equal to:

(1) 25 **(2)** $\frac{25}{2}$

(3) $\frac{15}{2}$ **(4)** $\frac{35}{2}$

Q. 16. Let a be a positive real number such that

$$\int_0^a e^{x-[x]}dx = 10e - 9$$

where $[x]$ is the greatest integer less than or equal to x. Then, a is equal to:

(1) $10 + \log_e 3$ **(2)** $10 - \log_e(1 + e)$

(3) $10 + \log_e 2$ **(4)** $10 + \log_e(1 + e)$

Q. 17. Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15$, $x \in R$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15$, $x \in R$ has a:

(1) local minimum at $x = -\frac{3}{4}$

(2) local maximum at $x = \frac{3}{4}$

(3) local minimum at $x = \frac{3}{4}$

(4) local maximum at $x = -\frac{3}{4}$

Q. 18. Let A = $[a_{ij}]$ be a 3 × 3 matrix, where

$$a_{ij} = \begin{cases} 1 & , \text{ if } \quad i = j \\ -x & , \text{ if } \quad |i-j| = 1 \\ 2x+1 & , \quad \text{otherwise} \end{cases}$$

Let a function f: R → R be defined as $f(x)$ = det(A). Then the sum of maximum and minimum values of f on R is equal to:

(1) $\frac{20}{27}$ **(2)** $-\frac{88}{27}$

(3) $-\frac{20}{27}$ **(4)** $\frac{88}{27}$

Q. 19. Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in$ **R** be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to:

(1) 24 **(2)** 18

(3) 45 **(4)** 36

Q. 20. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to:

(1) $\sim q \Rightarrow p$ **(2)** $p \Rightarrow q$

(3) $p \Rightarrow \sim q$ **(4)** $q \Rightarrow p$

Section B

Q. 21. Let T be the tangent to the ellipse E : $x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines $x = 1$ and $x = \sqrt{5}$ is $\sqrt{5}\alpha + \beta + \gamma \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha+\beta+\gamma|$ is equal to……..

Q. 22. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$ is ……

Q. 23. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is _________.

Q. 24. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a} + \vec{b} + \vec{c}$. Then, $36\cos^2 2\theta$ is equal to _____

Q. 25. Let P be a plane passing through the points (1, 0, 1), (1, –2, 1) and (0, 1, –2). Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{a}\cdot(\hat{i} + \hat{j} + 2\hat{j})$ = 2, then $(\alpha - \beta + \gamma)^2$ equals _________.

Q. 26. Let a, b, c, d be in arithmetic progression with common difference λ. If

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2,$$

then value of λ^2 is equal to _________.

Q. 27. If the value of $\lim_{x\to 0}\left(2 - \cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to______.

Q. 28. If the shortest distance between the lines $\vec{r_1} = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$, λ∈ R, α > 0 and $\vec{r_2} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$, μ ∈, R is 9, then α is equal to _____.

Q. 29. Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}\ (m+c)$ is equal to ______.

Q. 30. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3×3. If $B = [b_{ij}]$, then b_{13} is equal to _______.

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	1	Sine and Cosine Rule	Properties of Triangle
2	4	Classical Definition of Probability	Probability
3	1	Mean	Probability
4	4	Dot and Cross Product	Vector Algebra
5	4	Properties of Definite Integration	Definite Integration
6	3	Nature of Roots	Quadratic Equation
7	3	Exact Differential Equation	Differential Equation
8	2	Roots of Quadratic Equation	Quadratic Equation
9	2	Continuity of a Function	Continuity and Differentiability
10	3	Variable Separable Method	Differential Equation
11	4	Euler's Form	Complex Numbers and Quadratic Equation
12	3	Domain of Function	Functions
13	1	Domain	Inverse Trigonometric Function
14	3	General Term	Binomial Theorem
15	2	Tangent of Normal	Parabola
16	3	Properties of Definite Integration	Definite Integration
17	4	Local Extremum	Application of Derivative
18	2	Application of Derivative	Continuity and Differentiability
19	4	Symmetric and Skew Symmetric	Matrices Matrices and Determinants
20	2	Logical operation	Mathematical Reasoning
21	1.25	Area Under the Curve	Application of Integrals
22	21	Binomial Theorem	Binomial Theorem and Mathematical Induction
23	777	Combination	Permutation and Combination
24	4	Dot Product	Vector Algebra
25	81	Plane	Three Dimensional Geometry
26	1	Properties of Determinants	Matrices and Determinants
27	3	Limits	Limits and Derivatives
28	6	Shortest Distance Between Two Lines	Vector Algebra
29	34	Chord of Parabola	Parabola
30	910	Matrices	Matrices and Determinants

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ANSWERS WITH EXPLANATIONS

Mathematics

Section A

1. Option (1) is correct.

Given that, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$, R = 5 units

$\therefore \cos B = \frac{3}{5} \Rightarrow \sin B = \frac{4}{5}$

Now, $\frac{b}{\sin B} = 2R \Rightarrow b = 2R \sin B$

$= 2(5)\frac{4}{5}$

$\therefore \quad b = 8$

Now, By cosine rule

$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{3}{5}$$

$$\Rightarrow \frac{a^2+25-64}{10a} = \frac{3}{5}$$

$$\Rightarrow a^2 - 39 = 6a$$

$$\Rightarrow a^2 - 6a - 39 = 0$$

$$\Rightarrow a = \frac{6 \pm 8\sqrt{3}}{2}$$

$$\Rightarrow a = \frac{6 \pm 8\sqrt{3}}{2} = 3 \pm 4\sqrt{3}$$

$$\Rightarrow a = 3+4\sqrt{3}, \ a = 3-4\sqrt{3}$$

$$a = 3-4\sqrt{3} < 0$$

$\therefore \quad a = 3-4\sqrt{3}$ can not be possible (since length of side can not be negative

$\therefore \quad a = 3 + \sqrt{3}$

Now, area of $\Delta ABC = \frac{abc}{4R} = \frac{(3+4\sqrt{3})(8)(5)}{4(5)}$

$\therefore \quad \Delta = (6+8\sqrt{3})$ sq. units

Hint :

(i) The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

(ii) The cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$, $b^2 = a^2 + c^2 - 2ac \cos B$, $c^2 = a^2 + b^2 - 2ab \cos C$

(iii) $\Delta = \frac{abc}{R}$

Shortcut method:

(i) $\frac{b}{\sin B} = 2R$

(ii) $\cos B = \frac{(a^2+c^2-b^2)}{2ac}$

(iii) $\Delta = \frac{abc}{4R}$

2. Option (4) is correct.

In the given word (Examination)

$E \to 1, X \to 1, A \to 2, M \to 1,$
$O \to 1, T \to 1, N \to 2, I \to 2$

Total number of outcomes, $n(S) = \frac{11!}{2!2!2!}$

{where S is sample space}

Number of Favourable outcomes,

$$n(E) = \frac{10!}{2!2!2!}$$

Probability, $P(F) = \frac{n(E)}{n(S)} = \frac{\frac{10!}{2!2!2!}}{\frac{11!}{2!2!2!}} = \frac{1}{11}$

Hints:

(i) Probability, P = Number of Favourable Outcomes/Total Number of Outcomes

$P(E) = \frac{n(E)}{n(S)}$

Shortcut Method:
(i) Use formula $n!/(p_1!p_2!p_3!)$, since letters are repeating

3. **Option (1) is correct.**

Given, mean $\bar{x} = \frac{\Sigma x_i}{n} = 6.5$

$\Rightarrow \Sigma x_i = 6.5 \times 6 = 39$

Let remaining two number be x and y.

So, $18 + x + y = 39$

$\Rightarrow \quad x + y = 21 \quad ...(i)$

$\therefore \quad 10.25 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

$\Rightarrow \quad 10.25 = \frac{x^2 + y^2 + 4 + 16 + 25 + 49}{6} - (6.5)^2$

$\Rightarrow \quad 10.25 = \frac{x^2 + y^2 + 94}{6} - (6.5)^2$

$\Rightarrow \quad x^2 + y^2 = 221 \quad ...(ii)$

Solving (i) and (ii)

$\Rightarrow x^2 + (21 - x)^2 = 221$

$\Rightarrow 2x^2 - 42x + 220 = 0$

$\Rightarrow x^2 - 21x + 110 = 0$

$\Rightarrow (x - 10)(x - 11) = 0$

$\Rightarrow x = 10, 11$

So, $x = 10, y = 11$

Hints:

(i) Mean of $[x_1, x_2, ...x_n] = \frac{(x_1 + x_2 + ... + x_n)}{n}$

(ii) Variance σ^2.

$$= \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{x} = \frac{\sum_{i=1}^{n} x_i^2}{n} - \bar{x}^2$$

Shortcut method

$$10.25 = \frac{(x^2 + y^2 + 4 + 16 + 25 + 49)}{6} - (6.5)^2$$

Using y=21 - x, find x.
Then find y.

4. **Option (4) is correct.**

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = i(2) - j(2) + k(1) = 2\hat{i} - 2\hat{j} + \hat{k}$$

$\therefore \quad |\vec{a} \times \vec{b}| = 3$

Now, $|\vec{c} - \vec{a}|^2 = 8$

$\Rightarrow \quad |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c} = 8$

$\Rightarrow \quad |\vec{c}|^2 + 9 - 2|\vec{c}| = 8 \quad \left[\because |\vec{a}|^2 = 2^2 + 1^2 + (-2)^2 = 9\right]$

$\Rightarrow \quad |\vec{c}| - 2|\vec{c}|^2 + 1 = 0$

$\Rightarrow \quad |\vec{c}| = 1$

Now, $|(\vec{a} \times \vec{b}) \times \vec{c}| = |\vec{a} \times \vec{b}||\vec{c}| \sin\left(\frac{\pi}{6}\right)$

$= 3 \times 1 \times \frac{1}{2}$

$\Rightarrow \quad |(\vec{a} \times \vec{b}) \times \vec{c}| = \frac{3}{2}$

5. **Option (4) is correct.**

Given $\quad f(x) = \ln\left(\sqrt{1-x} + \sqrt{1+x}\right)$

$f(-x) = \ln\left(\sqrt{1-(-x)} + \sqrt{1-x}\right)$

$\Rightarrow \quad f(-x) = \ln\left(\sqrt{1+x} + \sqrt{1-x}\right) = f(x)$

$\Rightarrow \quad f(-x) = f(x)$

$\therefore \quad f$ is even function

Now, $I = \int_{-1}^{1} \ln\left(\sqrt{1-x} + \sqrt{1+x}\right) dx$

$\Rightarrow \quad I = 2\int_{0}^{1} \ln\left(\sqrt{1-x} + \sqrt{1+x}\right) dx$

Put, $x = \cos 2\theta \Rightarrow dx = -2 \sin 2\theta \, d\theta$

also $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Limits,

$x = 0, \theta = \frac{\pi}{4}$

$x = 1, \theta = 0$

$\Rightarrow \quad I = -4 \int_{\pi/4}^{0} \left[\ln\left\{(\sin\theta + \cos\theta)\sqrt{2}\right\}\right] \sin 2\theta d\theta$

$= 4 \int_{0}^{\pi/4} \left[\ln\left\{(\sin\theta + \cos\theta)\sqrt{2}\right\}\right] \sin 2\theta d\theta$

$= 4 \int_{0}^{\pi/4} \ln(\sin\theta + \cos\theta) \sin 2\theta d\theta$

$+ 4 \ln\sqrt{2} \int_{0}^{\pi/4} \sin 2\theta d\theta$

$$= 4\left[0+\frac{1}{2}\int_0^{\pi/4}(\cos\theta-\sin\theta)^2\,d\theta\right] + 4\ln\sqrt{2}\left(0+\frac{1}{2}\right)$$

$$= 4\left[0+\frac{1}{2}\int_0^{\pi/4}(1-\sin 2\theta)d\theta\right]+2\ln\sqrt{2}$$

$$= 2\left[\theta+\frac{\cos 2\theta}{2}\right]_0^{\pi/4}+\ln 2$$

$$= 2\left[\frac{\pi}{4}-\frac{1}{2}\right]+\ln 2$$

$\therefore \quad I = \frac{\pi}{2}-1+\ln 2$

6. Option (3) is correct.

Given that $x^2 + 2(a + 4)x - (5a - 64) > 0$

Comparing given quadratic equation with general form, $Ax^2 + Bx + C$

Here, $A = 1$, $B = 2(a + 4)$, $C = -(5a - 64)$

So, $D < 0$

$\Rightarrow B^2 - 4AC < 0$

$\Rightarrow 4(a + 4)^2 + 4(5a - 64) < 0$

$\Rightarrow (a + 4)^2 + (5a - 64) < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$\therefore \quad a = \frac{-13\pm\sqrt{169+192}}{2} = -16, 3$

So, $a \in (-16, 3)$

Since a is integer $\Rightarrow a = -5, -4, -3, -2, -1, 0, 1, 2$ as $a \in [-5, 30]$

$\therefore$ Required probability $= \frac{8}{36} = \frac{2}{9}$

7. Option (3) is correct.

Given differential equation

$\Rightarrow x\tan\left(\frac{y}{x}\right)dy = y\tan\left(\frac{y}{x}\right)dx - xdx$

$\Rightarrow \tan\left(\frac{y}{x}\right)(xdy-ydx) = -xdx$

$\Rightarrow \tan\left(\frac{y}{x}\right)\left(\frac{xdy-ydx}{x^2}\right) = \frac{-x}{x^2}dx$

$\Rightarrow \int\tan\left(\frac{y}{x}\right)\left(d\left(\frac{y}{x}\right)\right) = \int\frac{-1}{x}dx$

$\Rightarrow \ln|\sec(y/x)| = -\ln x + c$

$\Rightarrow \ln\left|x\sec\left(\frac{y}{x}\right)\right| = c$

Now, apply $y\left(\frac{1}{2}\right) = \frac{\pi}{6}$ in above

$\therefore \quad \ln\left|\frac{1}{2}\sec\left(\frac{\pi}{3}\right)\right| = c$

$\therefore \quad \ln\left|\frac{1}{2}\times 2\right| = c \quad \Rightarrow c = \ln 1 = 0$

$\therefore \quad \sec\left(\frac{y}{x}\right) = \frac{1}{x}$

$\therefore \quad y = x\sec^{-1}\left(\frac{1}{x}\right)$

So, required bounded area in upper half,

$$A = \int_0^{1/\sqrt{2}} x\sec^{-1}\left(\frac{1}{x}\right)dx = \int_0^{1/\sqrt{2}} x\cos^{-1}(x)dx$$

Using integration by parts

$$= \left|\left(\frac{x^2}{2}\cos^{-1}x\right)\right|_0^{\frac{1}{\sqrt{2}}} + \int_0^{\frac{1}{\sqrt{2}}}\frac{x^2}{2\sqrt{1-x^2}}$$

$$= \left(\frac{1}{4}\cdot\frac{\pi}{4}-0\right)+\frac{1}{2}\int_0^{\frac{1}{\sqrt{2}}}\frac{1-(1-x^2)}{\sqrt{1-x^2}}dx$$

$$= \frac{\pi}{16}+\frac{1}{2}\left[\left(\sin^{-1}x\right)_0^{\frac{1}{\sqrt{2}}} - \int_0^{\frac{1}{\sqrt{2}}}\sqrt{1-x^2}\,dx\right]$$

$$= \frac{\pi}{16}+\frac{1}{2}\left[\frac{\pi}{4}-\left\{\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right\}\Bigg|_0^{\frac{1}{\sqrt{2}}}\right]$$

$$= \frac{\pi}{16}+\frac{1}{2}\left[\frac{\pi}{4}-\left\{\frac{1}{4}+\frac{\pi}{8}\right\}\right]$$

$\therefore \quad$ Area $= \frac{\pi-1}{8}$

8. Option (2) is correct.

Given $x^2 + (3)^{1/4}x + 3^{1/2} = 0$

$\Rightarrow x^2 + \sqrt{3} = -3^{1/4}x$

Squaring both sides,

$\Rightarrow x^4 + 2\sqrt{3}x^2 + 3 = \sqrt{3}x^2$

$\Rightarrow x^4 + \sqrt{3}x^2 + 3 = 0$

$\Rightarrow x^4 + 3 = -\sqrt{3}x^2$

Now squaring both the sides again,

$\Rightarrow x^8 + 6x^4 + 9 = 3x^4$

$\Rightarrow x^8 + 3x^4 + 9 = 0$

Put $x = \alpha$, $\alpha^8 = -9 - 3\alpha^4$

$\therefore \quad \alpha^{12} = -9\alpha^4 - 3\alpha^8 = -9\alpha^4 - 3(-9 - 3\alpha^4) = 27$

Similarly $\beta^{12} = 27$

$\Rightarrow \quad \alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$

$= (27)^8 \times 26 + (27)^8 \times 26 = 52 \times (27)^8$

$= 52 \times 3^{24}$

9. Option (2) is correct.

Since, $f(x)$ is continuous at $x = 0$

So, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0+} f(x) = f(0)$

$\Rightarrow -1 = a - 1 = -1$

$\Rightarrow \quad a = 0$

Since, $f(x)$ is continuous at $x = 1$

So, $\lim_{x \to 1^-} f(x) = \lim_{x \to 1+} f(x) = f(1)$

$\Rightarrow a - 1 = 2 - b = 2 - b$

Put $a = 0$, so $0 - 1 = 2 - b$

$\Rightarrow \quad -3 = -b$

$\Rightarrow \quad b = 3$

So, the value of $a + b = 0 + 3 = 3$

10. Option (3) is correct.

Given differential equation

$$e^x\sqrt{1-y^2}\,dx + \left(\frac{y}{x}\right)dy = 0,\ y(1) = -1$$

$$\Rightarrow \quad e^x\sqrt{1-y^2}\,dx = \frac{-y}{x}dy$$

$$\Rightarrow \quad \frac{ydy}{\sqrt{1-y^2}} = -\int xe^x dx$$

$$\Rightarrow \quad \int\frac{-ydy}{\sqrt{1-y^2}} = \int xe^x dx$$

$$\Rightarrow \quad \sqrt{1-y^2} = e^x(x-1) + c$$

Given $x = 1, y = -1$

$\Rightarrow 0 = 0 + c \Rightarrow c = 0$

$\Rightarrow \sqrt{1-y^2} = e^x(x-1)$

At $\quad x = 3 \Rightarrow 1 - y^2 = (e^3 2)^2 \Rightarrow y^2 = 1 - 4e^6$

11. Option (4) is correct.

Given, $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \dfrac{3\pi}{2}$

$\Rightarrow |z||\omega| = 1$ and $\arg(z) = \arg(\omega) + \dfrac{3\pi}{2}$

Let $\omega = re^{i\theta} \Rightarrow z = \dfrac{1}{r}e^{i\left(\frac{3\pi}{2}+\theta\right)}$

$$\Rightarrow \quad \frac{1-2\bar{z}\omega}{1+3\bar{z}\omega} = \frac{1-2re^{i\theta}\frac{1}{r}e^{i\left(-\frac{3\pi}{2}-\theta\right)}}{1+3re^{i\theta}\frac{1}{r}e^{i\left(-\frac{3\pi}{2}-\theta\right)}}$$

$$= \frac{1-2e^{-i3\pi/2}}{1+3e^{-i3\pi/2}} = \frac{1-2i}{1+3i}$$

So, $\arg\left(\dfrac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right) = \arg\left(\dfrac{1-2i}{1+3i}\right)$

$= \tan^{-1}(-2) - \tan^{-1}(3)$

$= \dfrac{\pi}{4}$

12. Option (3) is correct.

$$f(x) = \sqrt{\frac{\|[x]\|-2}{\|[x]\|-3}}$$

$$\Rightarrow \quad \sqrt{\frac{\|[x]\|-2}{\|[x]\|-3}} \geq 0 \cap \|[x]\| - 3 \neq 0$$

Let $\quad t = \|[x]\|,\ t > 0$

$\Rightarrow \quad \sqrt{\dfrac{t-2}{t-3}} \geq 0 \Rightarrow \dfrac{t-2}{t-3} \geq 0$

$\Rightarrow \quad t \in (-\infty, 2] \cup (3, \infty) \cap t > 0$

$\Rightarrow \quad \|[x]\| \in [0, 2] \cup (3, \infty)$

$\Rightarrow \quad [x] \in (-\infty, -3) \cup [-2, 2] \cup (3, \infty)$

$\Rightarrow \quad x \in (-\infty, -3) \cup [-2, 3) \cup [4, \infty)$

So, $a = -3, b = -2, c = 3$

So, $a + b + c = -3 - 2 + 3 = -2$

13. Option (1) is correct.

Given, $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \dfrac{\pi}{4}$

As $\quad x^2 + x \geq 0$

$\Rightarrow x^2 + x + 1 \geq 1$

But $\quad x^2 + x + 1 \leq 1$ as $\sin^{-1}x \Rightarrow x \in [-1, 1]$

So, $\quad x^2 + x = 0$

$\Rightarrow \quad x = 0, -1$

put $x = 0, -1$ does not satisfies the original equation

$\Rightarrow \quad$ No solution

14. Option (3) is correct.

$(1 - x)^{101}(x^2 + x + 1)^{100}$

$= (1 - x)^{100}(x^2 + x + 1)^{100}(1 - x)$

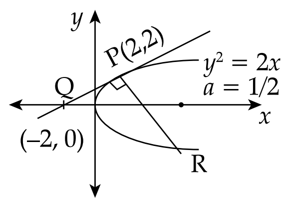
$= [(1-x)(1+x+x^2)]^{100}(1-x)$

$= (1-x^3)^{100}(1-x)$

$= (1-x)\,({}^{100}C_0 - {}^{100}C_1x^3 + {}^{100}C_2x^6 - \ldots\ldots + {}^{100}C_{84}x^{252} - {}^{100}C_{85}x^{255} + {}^{100}C_{86}x^{258} + \ldots.)$

$\therefore$ Coefficient of x^{256} is ${}^{100}C_{85} = {}^{100}C_{100-85} = {}^{100}C_{15}$

15. Option (2) is correct.



Tangent at P : $yy_1 = 2a(x + x_1)$

$y(2) = 2\left(\frac{1}{2}\right)(x+2)$

$\Rightarrow 2y = x + 2$

$\therefore$ Q = (–2, 0)

slope of tangent P $= \frac{1}{2}$

Normal at P : $y - 2 = -\frac{1}{\left(\frac{1}{2}\right)}(x-2)$

$\Rightarrow y - 2 = -2(x-2)$ $\quad [\because m_1m_2 = -1]$

$\Rightarrow y = 6 - 2x$

$\therefore$Now, solving with $y^2 = 2x \Rightarrow$ R $\left(\frac{9}{2}, -3\right)$

$$\therefore \text{Area}(\Delta PQR) = \frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -2 & 0 & 1 \\ \frac{9}{2} & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|2(0+3) - 2\left(-2-\frac{9}{2}\right) + 1(+6-0)\right|$$

$= \frac{25}{2}$ sq. units

16. Option (3) is correct.

Since, $a > 0, a \in R$

Let $n \le a < n + 1, n \in W$

$\therefore a = [a] + \{a\}$ .....(i)

where, $[a]$ is greatest integer factor

$\{a\}$ is fractional integer factor

$\therefore [a] = n$

Given: $\int_0^a e^{x-[x]}dx = 10e - 9$

$\Rightarrow \int_0^n e^{\{x\}}dx + \int_n^a e^{x-[x]}dx = 10e - 9$

$\Rightarrow n\int_0^1 e^x dx + \int_n^a e^{x-n}dx = 10e - 9$

$\Rightarrow n(e - e^0) + (e^{x-n} - e^{n-n}) = 10\,e - 9$

$\Rightarrow n = 0$ and $\{a\} = \ln 2$

Therefore, franequation (i)

$a = [a] + \{a\}$

$= 10 + \ln 2$

17. Option (4) is correct.

$f(x) = ax^2 + 6x - 15$

$\therefore \quad f'(x) = 2ax + 6$

For checking monotonic behaviour

$\Rightarrow \quad f'(x) = 0$

$\Rightarrow \quad x = -3/a$

According to the question $\frac{-3}{a} = \frac{3}{4} \Rightarrow a = -4$

Then $g(x) = -4x^2 - 6x + 15$

$g'(x) = -8x - 6$

For local maxima $g'(x) = 0$

$\Rightarrow \quad x = \frac{-3}{4}$

+ –

sign of $g'(x)$

$x = \frac{-3}{4}$

$\Rightarrow \quad x = \frac{-3}{4}$ is a point of local maxima

18. Option (2) is correct.

$$|A| = \begin{vmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{vmatrix}$$

$= 1(1 - x^2) + x(-x + x(2x + 1)) + (2x + 1)(x^2 - (2x + 1))$

$= 1 + x^2(2x + 1) + x^2(2x + 1) - (2x + 1)^2 - x^2 - x^2$

$\Rightarrow \quad f(x) = 4x^3 - 4x^2 - 4x$

$\Rightarrow \quad f'(x) = 12x^2 - 8x - 4$

$\Rightarrow \quad f'(x) = 4(3x^2 - 2x - 1) = 4(x-1)(3x+1)$

$$\underset{-1/3}{\overset{+ \qquad - \qquad +}{\longleftarrow\!\!\vert\!\!\longrightarrow}}\!\!\!\underset{1}{\vert} \quad \text{sign of } f'$$

$\Rightarrow \quad f(x)$ is maximum at $x = \dfrac{-1}{3}$ and

minimum at $x = 1$

Maximum value $= f\left(\dfrac{-1}{3}\right) = \dfrac{20}{27}$

Minimum value $f(1) = -4$

$\therefore$ Sum $= \dfrac{20}{27} - 4 = \dfrac{-88}{27}$

19. Option (4) is correct.

Since $A = \dfrac{1}{2}(A + A^T) + \dfrac{1}{2}(A - A^T)$

where $A + A^T$ is symmetric and $A - A^T$ is skew symmetric matrix.

$\Rightarrow P = \dfrac{1}{2}(A + A^T)$ and $Q = \dfrac{1}{2}(A - A^T)$

$\Rightarrow Q = \dfrac{1}{2}\begin{bmatrix} 0 & 3-a \\ a-3 & 0 \end{bmatrix}$

$\Rightarrow \quad \det(Q) = \dfrac{1}{4}(a-3)^2 = 9$

$\Rightarrow \quad (a-3)^2 = 36$

$\Rightarrow \quad a = 9$ or -3

Now, $P = \dfrac{1}{2}\begin{bmatrix} 4 & 3+a \\ a+3 & 0 \end{bmatrix}$

$\Rightarrow \quad \det(P) = \dfrac{-1}{4}(a+3)^2 = 36$ or 0

$\Rightarrow$ So, Modulus of all possible values of det (P) $= 36$

20. Option (2) is correct.

$(p \wedge \sim q) \Rightarrow (q \vee \sim p)$

$= (\sim p \vee q) \vee (q \vee \sim p)$

$= (\sim p \vee q) = (p \Rightarrow q)$

Section B

21. Correct answer is [1.25].

Equation of ellipse $x^2 + 4y^2 = 5$

or, $\dfrac{x^2}{5} + \dfrac{4y^2}{5} = 1$

Equation of tangent at P (1, 1) is $x + 4y = 5$

Now, area bounded by the required region

$$= \int_1^{\sqrt{5}} \left(\left(\frac{5-x}{4}\right) - \sqrt{\frac{5-x^2}{4}}\right) dx$$

$$= \left(\frac{5}{4}x - \frac{x^2}{8}\right)\Bigg|_1^{\sqrt{5}} - \frac{1}{2}\left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right]_1^{\sqrt{5}}$$

$$= \left(\frac{5\sqrt{5}}{4} - \frac{5}{8}\right) - \left(\frac{5}{4} - \frac{1}{8}\right) - \frac{1}{2}\left(0 + \frac{5\pi}{4}\right) + \frac{1}{2}\left(1 + \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)$$

$$= \frac{5}{4}\left(\sqrt{5} - 1\right) - \frac{1}{2} - \frac{5\pi}{8} + \frac{1}{2} + \frac{5}{4}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

$$= \frac{5\sqrt{5}}{4} - \frac{5}{4} - \frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

Comparing with the given condition

$\Rightarrow \quad \alpha = \dfrac{5}{4}, \beta = \dfrac{-5}{4}$ and $\gamma = \dfrac{-5}{4}$

$\Rightarrow \quad |\alpha + \beta + \gamma| = \dfrac{5}{4} = 1.25$

22. Correct answer is [21].

General term of $\left(2^{\frac{1}{2}} + 5^{\frac{1}{6}}\right)^{120}$ is

Given by $T_{r+1} = {}^{120}C_r \left(2^{\frac{1}{2}}\right)^{120-r} \left(5^{\frac{1}{6}}\right)^r$

For rational term, r should be a multiple of 6 (*i.e.*) $r \in \{0, 6, 12, 18, \ldots, 120\}$

$\therefore$ 21 rational terms are there in the expansion $\left(2^{\frac{1}{2}} + 5^{\frac{1}{6}}\right)^{20}$

23. Correct answer is [777].

Case I : Team consist 5 batsman, 5 bowlers and 1 wicket keeper then, number of ways

$= {}^6C_5 \times {}^7C_5 \times {}^2C_1 = 6 \times 21 \times 2 = 252$

Case II : 4 bowlers, 6 batsman and 1 wicket keepers

$= {}^6C_4 \times {}^7C_6 \times {}^2C_1 = 15 \times 7 \times 2 = 210$

Case III : 4 bowlers, 5 batsman and 2 wicket keepers

$= {}^6C_4 \times {}^7C_5 \times {}^2C_2 = 15 \times 21 \times 1 = 315$

Total $= 252 + 210 + 315 = 777$

24. Correct answer is [4].

$$|\vec{a}+\vec{b}+\vec{c}|^2=(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})$$

$$=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{c})$$

$$=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+0$$

$$\therefore\ |\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2=3k^2$$

$$\therefore\ |\vec{a}+\vec{b}+\vec{c}|=\sqrt{3}\,k$$

Now, $\vec{a}.(\vec{a}+\vec{b}+\vec{c})=|\vec{a}||\vec{a}+\vec{b}+\vec{c}|\cos\theta$

$$|\vec{a}.|^2+\vec{a}.\vec{b}+\vec{a}.\vec{c}=|\vec{a}||\vec{a}+\vec{b}+\vec{c}|\cos\theta$$

$\Rightarrow k^2+0=k\times\sqrt{3}\ k\cos\theta$

$$\Rightarrow\quad \cos\theta=\frac{1}{\sqrt{3}}\Rightarrow\cos 2\theta=2\cos^2\theta-1$$

$$\Rightarrow\quad \cos 2\theta=\frac{-1}{3}\quad\Rightarrow\cos^2 2\theta=\frac{1}{9}$$

$$\Rightarrow\quad 36\cos^2 2\theta=4$$

25. Correct answer is [81].

Equation of plane P is

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix}=0$$

or,
$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 0 & 2 & 0 \\ 1 & -1 & 3 \end{vmatrix}=0$$

$\Rightarrow\quad 6(x-1)+(z-1)(-2)=0\Rightarrow 3x-z-2=0$

Normal vector to the plane P is $\vec{n}=3\hat{i}-\hat{k}$

Now, $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ is perpendicular to $\vec{n}$

$\Rightarrow\quad \vec{a}\cdot\vec{n}=0\Rightarrow 3\alpha-\gamma=0.$ ...(i)

Also $\vec{a}$. is perpendicular to $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

$\Rightarrow\quad \vec{a}\cdot\vec{b}=0\Rightarrow\alpha+2\beta+3\gamma=0$ ...(ii)

And $\vec{a}.(\hat{i}+\hat{j}+2\hat{k})=2$

$\Rightarrow\quad \alpha+\beta+2\gamma=2$ ...(iii)

by solving (i), (ii) and (iii)

$\Rightarrow\quad \alpha=1,\beta=-5,\gamma=3\Rightarrow(\alpha-\beta+\gamma)^2=81$

26. Correct answer is [1].

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix}=2$$

Given, a, b, c and d are in AP

$(b-a)=(c-b)=(d-c)$

$\Rightarrow a+c=2b,\ b+d=2c$

$C_2\to C_2-C_3$

$$\Rightarrow\quad \begin{vmatrix} x-2\lambda & \lambda & x+a \\ x-1 & \lambda & x+b \\ x+2\lambda & \lambda & x+c \end{vmatrix}=2$$

$R_2\to R_2-R_1,\ R_3\to R_3-R_1$

$$\Rightarrow\quad \begin{vmatrix} x-2\lambda & \lambda & x+a \\ 2\lambda-1 & 0 & \lambda \\ 4\lambda & 0 & 2\lambda \end{vmatrix}=2$$

$\Rightarrow\quad -\lambda[(2\lambda-1)2\lambda-4\lambda^2)=2$

$\Rightarrow\quad 2\lambda^2=2$

$\lambda^2=1$

27. Correct answer is [3].

$$\lim_{x\to0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$$

From indeterminate form 1^∞

$$=e^{\lim_{x\to0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)(x+2)}$$

$$=e^{\lim_{x\to0}\left(\frac{1-\cos^2 x.\cos 2x}{x^2(1+\cos x\sqrt{\cos 2x})}\right)(x+2)}$$

$$=e^{\lim_{x\to0}\frac{(1-(1-\sin^2 x)(1-2\sin^2 x))(x+2)}{x^2(1+\cos x\sqrt{\cos 2x})}}$$

$$=e^{\lim_{x\to0}\frac{(3\sin^2 x-2\sin^4 x)(x+2)}{x^2(1+\cos x\sqrt{\cos 2x})}}$$

$$=e^{\lim_{x\to0}\left(\frac{\sin^2 x}{x^2}\right)\frac{(3-2\sin^2 x)(x+2)}{(1+\cos x\sqrt{\cos 2x})}}$$

$$=e^{\frac{3\times2}{2}}=e^3$$

$\therefore\ a=3$

28. Correct answer is [6].

Given equation of lines

$$\vec{r}_1=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$$

$$\vec{r}_2=-4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$$

$$\text{Shortest distance}=\left|\frac{(\vec{a}_1-\vec{a}_2)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$\therefore\quad 9=\left|\frac{((\alpha+4)\hat{i}+2\hat{j}+3\hat{k}).(8\hat{i}+8\hat{j}+4\hat{k})}{\sqrt{64+64+16}}\right|$$

$$\Rightarrow \quad \left|\frac{8(\alpha+4)+16+12}{12}\right| = 9$$

$\therefore \quad \alpha = 6,\ \text{as}\ \alpha > 0$

29. Correct answer is [34].

Given equation of parabola $y^2 = -64x$

Focus $= (-16, 0)$

Let focal chord $\quad y = mx + c$

$\Rightarrow \quad c = 16\,m \qquad$ ...(i)

If $y = mx + c$ is tangent to

$(x + 10)^2 + y^2 = 4$

$\Rightarrow \quad y = m(x + 10) \pm 2\sqrt{1+m^2}$

$\therefore \quad c = 10\,m \pm 2\sqrt{1+m^2}$

So, $\quad 16m = 10m \pm 2\sqrt{1+m^2}$

.......(from (i))

$\Rightarrow \quad 6m = \pm 2\sqrt{1+m^2}$

$\Rightarrow \quad 3m = \pm\sqrt{1+m^2}$

Squaring both sides,

$\Rightarrow \quad 9m^2 = 1 + m^2$

$\Rightarrow \quad 8m^2 = 1$

$\Rightarrow \quad m = \dfrac{1}{2\sqrt{2}},\ (m > 0)$ and $c = \dfrac{8}{\sqrt{2}}$

$$\therefore \quad 4\sqrt{2}(m+c) = 4\sqrt{2}\left(\frac{1}{2\sqrt{2}} + \frac{8}{\sqrt{2}}\right)$$

$$= 4\sqrt{2}\left(\frac{17}{2\sqrt{2}}\right) = 34$$

30. Correct answer is [910].

Let $\quad A = I + C$

Where $\quad C = \begin{vmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{vmatrix}$

$$C^2 = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$C^3 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

So, $\quad A^2 = (I + C)^2 = I + 2C + C^2$

$A^3 = A^2 \cdot A = I + 3C + 3C^2$

$A^4 = I + 4C + 6C^2$

$A^5 = I + 5C + 10C^2$

So, $\quad A^n = I + n\,C + \dfrac{n(n-1)}{2}C^2$

$A^{20} = I + 20C + 190C^2$

$\therefore \quad A^7 = I + 7C + 21C^2$

$\therefore \quad B = 7A^{20} - 20A^7 + 2I$

$B = 7(I + 20C + 190\,C^2)$

$-20(I + 7\,C + 21\,C^2)$

$+ 2I$

$\therefore \quad B = -11\,I + 910\,C^2$

$$= \begin{vmatrix} -11 & 0 & 910 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{vmatrix}$$

$\therefore \quad b_{13} = 910$

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