JEE (Main) PHYSICS SOLVED PAPER

2022

29^{th} June Shift 1

Time : 1 Hour **Total Marks : 100**

General Instructions :

1. *In Physics Section, there are 30 Questions (Q. no. 1 to 30) having Section A and B.*
2. *Section A consists of 20 multiple choice questions & Section B consists of 10 numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
3. *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
5. *Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.*
6. *All calculations / written work should be done in the rough sheet is provided with Question Paper.*

Physics

Section A

Q. 1. Two balls *A* and *B* are placed at the top of 180 m tall tower. Ball *A* is released from the top at $t = 0$ s. Ball B is thrown vertically down with an initial velocity 'u' at $t = 2$ s. After a certain time, both balls meet 100 m above the ground. Find the value of 'u' in ms^{-1} . [use $g = 10\ ms^{-2}$] :

(A) 10 **(B)** 15
(C) 20 **(D)** 30

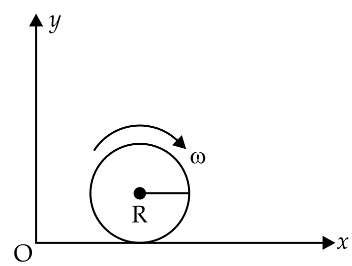
Q. 2. A body of mass *M* at rest explodes into three pieces, in the ratio of masses 1:1:2. Two smaller pieces fly off perpendicular to each other with velocities of 30 ms^{-1} and 40 ms^{-1} respectively. The velocity of the third piece will be:

(A) 15 ms^{-1} **(B)** 25 ms^{-1}
(C) 35 ms^{-1} **(D)** 50 ms^{-1}

Q. 3. The activity of a radioactive material is 2.56×10^{-3} Ci. If the half life of the material is 5 days, after how many days the activity will become 2×10^{-5} Ci ?

(A) 30 days **(B)** 35 days
(C) 40 days **(D)** 25 days

Q. 4. A spherical shell of 1 kg mass and radius R is rolling with angular speed ω on horizontal plane (as shown in figure). The magnitude of angular momentum of the shell about the origin O is $\frac{a}{3} R^2 \omega$. The value of a will be :



(A) 2 **(B)** 3
(C) 5 **(D)** 4

Q. 5. A cylinder of fixed capacity of 44.8 litres contains helium gas at standard temperature and pressure. The amount of heat needed to raise the temperature of gas in the cylinder by 20.0°C will be : (**Given:** gas constant $R = 8.3 JK^{-1} mol^{-1}$)

(A) 249 J **(B)** 415 J
(C) 498 J **(D)** 830 J

Q. 6. A wire of length L is hanging from a fixed support. The length changes to L_1 and L_2 when masses 1 kg and 2 kg are suspended respectively from its free end. Then the value of L is equal to :

(A) $\sqrt{L_1 L_2}$ **(B)** $\frac{L_1 + L_2}{2}$
(C) $2 L_1 - L_2$ **(D)** $3L_1 - 2L_2$

Q. 7. Given below are two statements: one is labelled as Assertion **A** and the other is labelled as Reason **R.**

Assertion A : The photoelectric effect does not takes place, if the energy of the incident radiation is less than the work function of a metal.

Reason R : Kinetic energy of the photoelectrons is zero, if the energy of the incident radiation is equal to the work function of a metal.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(B) Both **A** and **R** are correct but **R** is not the correct explanation of **A**

(C) **A** is correct but **R** is not correct

(D) **A** is not correct but **R** is correct

Q. 8. A particle of mass 500 gm is moving in a straight line with velocity $v = bx^{5/2}$. The work done by the net force during its displacement from $x = 0$ to $x = 4$ m is : (Take $b = 0.25\ \text{m}^{-3/2}\ \text{s}^{-1}$).

(A) 2 J **(B)** 4 J

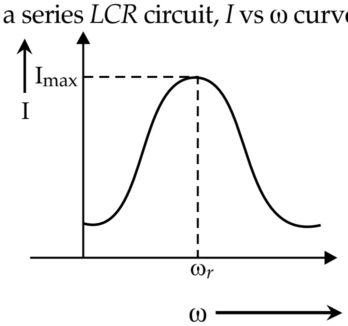
(C) 8 J **(D)** 16 J

Q. 9. A charge particle moves along circular path in a uniform magnetic field in a cyclotron. The kinetic energy of the charge particle increases to 4 times its initial value. What will be the ratio of new radius to the original radius of circular path of the charge particle

(A) 1 : 1 **(B)** 1 : 2

(C) 2 : 1 **(D)** 1 : 4

Q. 10. For a series *LCR* circuit, *I* vs ω curve is shown :



(a) To the left of ω_r, the circuit is mainly capacitive.

(b) To the left of ω_r, the circuit is mainly inductive.

(c) At ω_r, impedance of the circuit is equal to the resistance of the circuit.

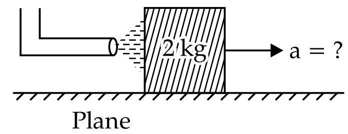
(d) At ω_r, impedance of the circuit is 0 .

Choose the most appropriate answer from the options given below :

(A) (a) and (d) only **(B)** (b) and (d) only

(C) (a) and (c) only **(D)** (b) and (c) only

Q. 11. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1kgs^{-1} and at a speed of $10\ \text{ms}^{-1}$. Then, the initial acceleration of the block, in ms^{-2}, will be :



(A) 3 **(B)** 6

(C) 5 **(D)** 4

Q. 12. In Van der Waals equation $\left[P + \frac{a}{V^2}\right][V - b] =$ RT ; P is pressure, V is volume, R is universal gas constant and T is temperature. The ratio of constants $\frac{a}{b}$ is dimensionally equal to:

(A) $\frac{P}{V}$ **(B)** $\frac{V}{P}$

(C) PV **(D)** PV^3

Q. 13. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A} + \vec{B}$ is equal to two times the magnitude of $\vec{A} - \vec{B}$, then the angle between $\vec{A}$ and $\vec{B}$ will be :

(A) $\sin^{-1}\left(\frac{3}{5}\right)$ **(B)** $\sin^{-1}\left(\frac{1}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{5}\right)$ **(D)** $\cos^{-1}\left(\frac{1}{3}\right)$

Q. 14. The escape velocity of a body on a planet '*A*' is $12\ \text{kms}^{-1}$. The escape velocity of the body on another planet '*B*', whose density is four times and radius is half of the planet '*A*', is :

(A) $12\ \text{kms}^{-1}$ **(B)** $24\ \text{kms}^{-1}$

(C) $36\ \text{kms}^{-1}$ **(D)** $6\ \text{kms}^{-1}$

Q. 15. At a certain place the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.5 G. The earth's total magnetic field (in G), at that certain place, is :

(A) $\frac{1}{\sqrt{3}}$ **(B)** $\frac{1}{2}$

(C) $\sqrt{3}$ **(D)** 1

Q. 16. A longitudinal wave is represented by $x = 10 \sin 2\pi\left(nt - \frac{x}{\lambda}\right)$ cm. The maximum particle velocity will be four times the wave velocity if the determined value of wavelength is equal to :

(A) 2π **(B)** 5π

(C) π **(D)** $\frac{5\pi}{2}$

Q. 17. A parallel plate capacitor filled with a medium of dielectric constant 10, is connected across a battery and is charged. The dielectric slab is replaced by another slab of dielectric constant 15. Then the energy of capacitor will :

(A) increase by 50% **(B)** decrease by 15%
(C) increase by 25% **(D)** increase by 33%

Q. 18. A positive charge particle of 100 mg is thrown in opposite direction to a uniform electric field of strength 1×10^5 NC^{-1}. If the charge on the particle is 40 μC and the initial velocity is 200 ms^{-1}, how much distance it will travel before coming to the rest momentarily :

(A) 1 m (B) 5 m
(C) 10 m (D) 0.5 m

Q. 19. Using Young's double slit experiment, a monochromatic light of wavelength 5000Å produces fringes of fringe width 0.5 mm. If another monochromatic light of wavelength 6000Å is used and the separation between the slits is doubled, then the new fringe width will be :

(A) 0.5 mm **(B)** 1.0 mm
(C) 0.6 mm **(D)** 0.3 mm

Q. 20. Only 2% of the optical source frequency is the available channel bandwidth for an optical communicating system operating at 1000 nm. If an audio signal requires a bandwidth of 8 kHz, how many channels can be accommodated for transmission :

(A) 375×10^7 **(B)** 75×10^7
(C) 375×10^8 **(D)** 75×10^9

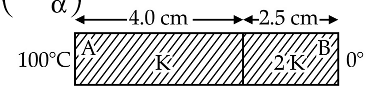
Section B

Q. 21. Two coils require 20 minutes and 60 minutes respectively to produce same amount of heat energy when connected separately to the same source. If they are connected in parallel arrangement to the same source; the time required to produce same amount of heat by the combination of coils, will be ______ min.

Q. 22. The intensity of the light from a bulb incident on a surface is 0.22 W/m^2. The amplitude of the magnetic field in this light-wave is ______ $\times 10^{-9}$T

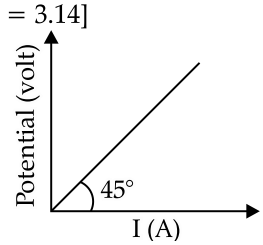
(**Given:** Permittivity of vacuum $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} – m^{-2}, speed of light in vacuum $c = 3 \times 10^8$ ms^{-1})

Q. 23. As per the given figure, two plates A and B of thermal conductivity K and 2 K are joined together to form a compound plate. The thickness of plates are 4.0 cm and 2.5 cm respectively and the area of cross-section is 120 cm^2 for each plate. The equivalent thermal conductivity of the compound plate is $\left(1+\frac{5}{\alpha}\right)K$, then the value of α will be ——.



Q. 24. A body is performing simple harmonic with an amplitude of 10 cm. The velocity of the body was tripled by air Jet when it is at 5 cm from its mean position. The new amplitude of vibration is $\sqrt{x}$ cm. The value of x is ——.

Q. 25. The variation of applied potential and current flowing through a given wire is shown in figure. The length of wire is 31.4 cm. The diameter of wire is measured as 2.4 cm. The resistivity of the given wire is measured as x × 10^{-3} Ωcm. The value of x is _______. [Take π = 3.14]



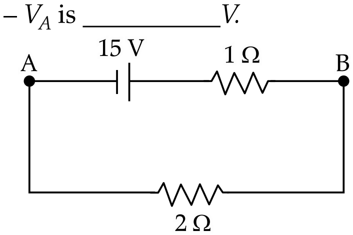
Q. 26. 300 cal. of heat is given to a heat engine and it rejects 225 cal. of heat. If source temperature is 227°C, then the temperature of sink will be ______ °C

Q. 27. $\sqrt{d_1}$ and $\sqrt{d_2}$ are the impact parameters corresponding to scattering angles 60° and 90° respectively, when an α particle is approaching a gold nucleus. For $d_1 = xd_2$, the value of x will be ______

Q. 28. A transistor is used in an amplifier circuit in common emitter mode. If the base current changes by 100μA, it brings a change of 10 mA in collector current. If the load resistance is 2kΩ and input resistance is 1kΩ, the value of power gain is x × 104 . The value of x is .

Q. 29. A parallel beam of light is allowed to fall on a transparent spherical globe of diameter 30cm and refractive index 1.5. The distance from the centre of the globe at which the beam of light can converge is ______mm.

Q. 30. For the network shown below, the value of $V_B - V_A$ is ___________V.



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Answer Key

Q. No.	Answer	Topic Name	Chapter Name
Section (A)			
1	D	Motion in a Straight Line	Kinematics
2	B	Laws of Motion	Laws of Motion
3	B	Nuclei	Atoms and Nuclei
4	C	Motion of System of Particles and Rigid Body	Motion of System of Particles and Rigid Body
5	C	Behaviour of Perfect Gas and Kinetic Theory	Behaviour of Perfect Gas and Kinetic Theory
6	C	Laws of Motion	Laws of Motion
7	B	Dual Nature of Matter and Radiation	Dual Nature of Matter and Radiation
8	D	Work, Energy and Power	Work, Energy and Power
9	C	Magnetic Effect of Current	Magnetic Effects of Current and Magnetism
10	C	Alternating Currents	Electromagnetic Induction and Alternating Current
11	C	Laws of Motion	Laws of Motion
12	C	Behaviour of Perfect Gas and Kinetic Theory	Behaviour of Perfect Gas and Kinetic Theory
13	C	Concept of Vector and Motion in a Plane	Kinematics
14	A	Gravitation	Gravitation
15	A	Magnetism and Magnetic Matter	Magnetic Effects of Current and Magnetism
16	B	Waves	Oscillations and Waves
17	A	Electrostatic Potential and Capacitance	Electrostatics
18	D	Electric Charges and Fields	Electrostatics
19	D	Wave Optics	Optics
20	B	Communication	Communication
Section (B)			
21	15	Current Electricity	Current Electricity
22	43	Electromagnetic Waves	Electromagnetic Waves
23	21	Thermal Properties of Matter	Properties of Bulk Matter
24	700	Oscillations	Oscillations and Waves
25	144	Current Electricity	Current Electricity
26	102	Thermodynamics	Thermodynamics
27	3	Atoms	Atoms and Nuclei
28	2	Electronic Devices	Electronic Devices
29	225	Ray Optics and Optical Instruments	Optics
30	10	Current Electricity	Current Electricity

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ANSWERS WITH EXPLANATIONS

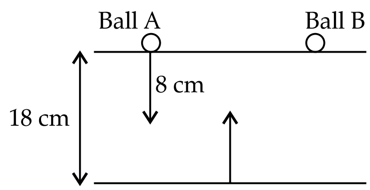
Physics

Section A

1. Option (D) is correct.

Explanation: As ball A is dropped from tower

$u = 0$ m/s



As the meeting point lies 100 m above ground, displacement of ball will be 80 m.

For ball A

$u = 0$, S = 80 m, $a = +g = +10$ m/s^2, time = t_1

$$S = ut + \frac{1}{2}at^2$$

$$80 = 0 + \frac{1}{2} \times 10 \times t_1^2$$

$$\frac{160}{10} = t_1^2$$

$$\Rightarrow \quad t_1 = 4\ s$$

As ball B is thrown after 2 seconds after release of A. Thus, time available for ball B is 2 seconds to cover a distance of 80 m.

Let speed be 'u' m/s, $t_2 = 4 - 2 = 2\ s$, S = 80 m, $a = +g = +10$ m/s^2

$$\therefore \quad 80 = u \times 2 + \frac{1}{2} \times 10 \times (2)^2$$

$$80 = u + 20$$

$$2u = 60$$

$$\Rightarrow \quad u = 30 \text{ m/s}$$

Hint :

(i) As the motion is under gravity i.e., free fall it can be assumed to be uniformly accelerated motion.

(ii) Use $S = ut + \frac{1}{2}at^2$ two times, first for ball A and then for ball B.

(iii) Solve for initial velocity of ball B.

Shortcut:

As distance covered is same for both the balls, then

$$\frac{1}{2}gt^2 = u\,(t-2) + \frac{1}{2}g\,(t-2)^2 = 80 \quad ...(1)$$

$$\frac{1}{2}gt^2 = 80 \Rightarrow t = 4\ s$$

By substituting value of t in equation 1, we get:

$$u\,(\text{t} - 2) + \frac{1}{2}g\,(t-2)^2 = 80$$

$$u\,(4-2) + \frac{1}{2}g\,(4-2)^2 = 80$$

$$u = 30 \text{ m/s}$$

2. Option (B) is correct.

Explanation: In the given problem a body of mass M explodes into three pieces of mass ratio 1:1:2

Thus, the mass of fragments will be x, x, $2x$

Hence, $M = x + x + 2x = 4x$ kg

As in the process of explosion no external forces are involved and explosion occurs due to internal forces. Thus, momentum of the system will be conserved.

Initially M is at rest

$p_{\text{initial}} = p_{\text{final}}$

By law of conservation of momentum x

$$M \times 0 = \frac{M}{4} \times 30\hat{i} + \frac{M}{4} \times 40\hat{j} + \frac{2M}{4}\vec{v}$$

Where $\vec{v}$ is the velocity of the third fragment.

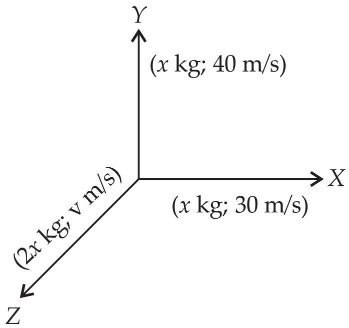
$$\frac{M}{2}\vec{v} = -\frac{M}{4}(30\hat{i} + 40\hat{j})$$

$$\vec{v} = -15\hat{i} - 20\hat{j}$$

Thus, magnitude of $\vec{v} = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{(-15)^2 + (-20)^2}$$

$$|\vec{v}| = \sqrt{625} = 25 \text{ m/s}$$



Hint **:**

(i) As the explosion process does not include any external forces. Hence momentum of the system will be conserved.

(ii) Apply law of conservation of momentum

Shortcut:

Assume mass of the body to be 4 kg

⇒ Mass of fragments will be 1 kg, 1kg and 2 kg respectively.

⇒ Momentum of 3rd fragment = $-(\vec{p_1}+\vec{p_2})$

$$\vec{v_3} = \frac{-(1\times 30\hat{i}+1\times 40\hat{j})}{2} = -15\hat{i}-20\hat{j}$$

$$(\vec{v}_3) = \sqrt{(-15)^2+(-20)^2} = \sqrt{625} = 25 \text{ m/s}$$

3. Option (B) is correct.

Explanation: Given, at $t = 0 \Rightarrow$ Activity of Radioactive Sample = 2.56×10^{-3} Ci

Thus, $R_o = 2.56 \times 10^{-3}$ Ci

Half life of sample, $T_{½} = 5$ days

As, Radioactive decay is of first order reaction. Thus, Rate of decay or activity decreases exponentially.

By Radioactive Decay law,

$R = R_o e^{-\lambda t}$

$\Rightarrow 2\times 10^{-5} = 2.56 \times 10^{-3}\, e^{-\lambda t}$

Where, R ⇒ Activity at time t

λ ⇒ Activity constant of Radioactive sample

$$\lambda \Rightarrow \frac{ln 2}{T_{1/2}}$$

Taking logarithm on both sides

$ln\,(2 \times 10^{-5}) = ln\,(2.56 \times 10^{-3}) + ln\,(e^{-\lambda t})$

$ln\,(2 \times 10^{-5}) - ln\,(2.56 \times 10^{-3}) = -\lambda t$

$$ln\left(\frac{2\times 10^{-5}}{2.56\times 10^{-3}}\right) = -\lambda t$$

$$ln\left(\frac{1}{128}\right) = -\lambda t$$

$$-\,ln\,128 = -\lambda t$$

$$\Rightarrow \quad ln\,2^7 = \frac{ln 2}{T_{1/2}}t$$

$$\Rightarrow \quad 7 ln\,2 = \frac{ln 2}{T_{1/2}}t$$

$$\Rightarrow \quad t = 7\,T_{½}$$

$$\Rightarrow \quad t = 7 \times 5 = 35 \text{ days}$$

Hint **:**

(i) Apply Law of Radioactive Decay

(ii) Use $R = R_o e^{-\lambda t}$

Shortcut:

We can calculate number of half lives taken for activity to change from 2.56×10^{-3} to 2×10^{-5} Ci By relation:

$$\frac{R}{R_o} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{2\times 10^{-5}}{2.56\times 10^{-3}}\right) = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{128}\right) = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^n$$

As, $\quad n = \dfrac{T}{T_{1/2}}$

Thus, $\quad n = 7$

Hence, $\quad T = n \times T_{1/2}$

$= 7 \times 5$

$= 35$ days

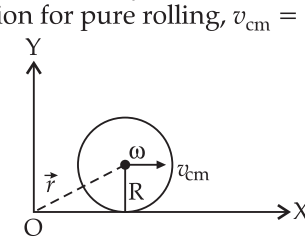
4. Option (C) is correct.

Explanation: Given: Mass of hollow sphere = 1 kg

Radius of sphere = R

As sphere is in pure rolling motion, then sphere will be rotating about its centre of mass with angular speed ω and its centre of mass will also perform translatory motion with velocity= v_{cm}

Condition for pure rolling, $v_{cm} = R\omega$



⇒ Total Angular momentum of Rolling Sphere = Angular momentum due to rotation about it's centre of mass + Moment of linear momentum possessed by centre of mass about origin.

$\Rightarrow L_{\text{net}} = L_{\text{cm}} + \vec{r} \times (M\vec{v}_{\text{cm}})$

$\Rightarrow L_{\text{net}} = I\omega + Mv_{\text{cm}} \times r_{\perp ar}$

$\Rightarrow L_{\text{net}} = I\omega + Mv_{\text{cm}}R$

Where $r_{\perp ar}$ is the perpendicular distance between centre of mass and line passing from origin

$\therefore r_{\perp ar} = R$

Moment of inertia of hollow sphere about its centre of mass $= \frac{2}{3}MR^2$

$$L_o = \frac{2}{3}MR^2\omega^2 + 1 \times R\omega \times R$$

$$L_o = \frac{2}{3} \times 1 \times R^2\omega + R^2\omega$$

$$L_o = R^2\omega\left[\frac{2}{3}+1\right]$$

$$L_o = \frac{5}{3}R^2\omega$$

So, by comparing it with given value, we get, $a = 5$.

Hint: 1. Net Angular momentum of sphere about origin = Angular momentum due to Rotation about its centre of mass + Moment of linear momentum possessed by centre of mass

$$L_o = I\omega + \vec{r} \times (M\vec{v}_{\text{cm}})$$

2. Use condition of pure rolling i.e., $v_{\text{cm}} = R\omega$

Shortcut: $L_o = Mv_{\text{cm}}R + I\omega$

$$= 1 \times R\omega R + \frac{2}{3}MR^2\omega^2$$

$$= \omega R^2 + \frac{2}{3} \times 1\,\omega R^2$$

$$= \frac{5}{3}\omega R^2$$

Thus, $a = 5$

5. Option (C) is correct.

Explanation: As the cylinder is of fixed capacity it means its volume is fixed.

Hence, it is an Isochoric process.

Helium is at S.T.P. conditions (given).

By Avogadro Hypothesis, one mole of gas occupies 22.4 L volume at S.T.P. condition.

⇒ Thus, at S.T.P. 44.8 L volume will be occupied by 2 moles of Helium Gas.

⇒ As, Helium is monoatomic gas, its degree of freedom will be 3.

For Isochoric Process

$Q = \Delta U$ where Q is the heat transfer $\Delta U \Rightarrow$ change in internal energy of gas $\Delta U = nC_V\Delta T$ wgere C_V is specific heat at constant volume

$$\Rightarrow \quad C_V = \frac{fR}{2} = \frac{3R}{2} \text{ for monoatomic gas}$$

(where, f = degree of freedom)

Given: $\Delta T = 20°C$

Thus, $Q = \Delta U = 2 \times \frac{3}{2} R \times 20 = 60\,R$

$\Rightarrow \quad Q = 60 \times 8.3 = 498$ J

Hint **:**

(i) By Avogadro Hypothesis, calculate number of moles of Helium gas.

(ii) Capacity of cylinder is fixed, so gas follows constant volume process.

(iii) Apply $Q = nC_V\Delta T$, because work done is zero in isochoric process.

Shortcut **:**

By first law of thermodynamics,

$$Q = \Delta U + w \qquad ...(i)$$

As container is rigid so, change in volume = 0

Thus, work done = 0

$$Q = \Delta U = nC_V\Delta T$$

At S.T.P. condition 1 mole of gas occupies 22.4 L volume, so 2 moles of Helium will occupy 44.8 L volume.

As Helium is monoatomic gas, so its degree of freedom = 3

Thus, $C_V = \frac{f}{2} = \frac{3R}{2}$

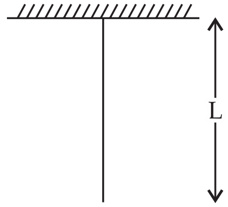
$$\Rightarrow \quad Q = 2 \times \frac{3R}{2} \times 20$$

$$= 60\,R = 60 \times 8.3 = 498 \text{ J}$$

6. Option (C) is correct.

Explanation: Given: When 1 kg mass is attached to wire the final length becomes L_1 and when 2 kg mass is attached to wire, the final length of wire becomes L_2.

Let the Young's modulus of wire material be Y, Then

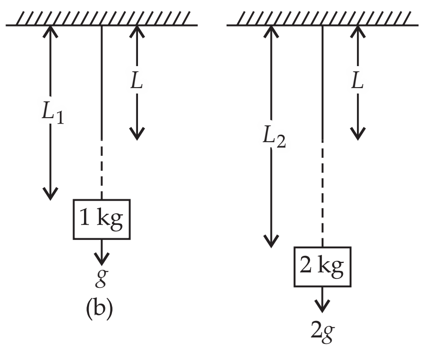


(a)

By Hooke's Law of elasticity $\Rightarrow \frac{F}{A} = Y\frac{\Delta l}{l}$

Let area of cross-section of wire be A
original length of wire = L
Case-I → When 1 kg load is attached to wire change in length = $L_1 - L$
Using Hooke's Law

$$F_1 = mg = 1 \times g \Rightarrow \frac{1 \times g}{A} = Y\left(\frac{L_1 - L}{L}\right) \quad ...(1)$$



Length of wire
(a) in normal condition
(b) When 1 kg mass is suspended
(c) When 2 kg mass is suspended
Case-II → When 2 kg load is attached to wire change in length = $L_2 - L$
Using Hooke's Law

$$F_2 = mg = 2 \times g$$

$$\Rightarrow \frac{2 \times g}{A} = Y\left(\frac{L_2 - L}{L}\right) \quad ...(2)$$

Dividing equation (1) by (2)

$$\frac{1}{2} = \frac{L_1 - L}{L_2 - L}$$

$$L_2 - L = 2L_1 - 2L$$

$$\boxed{L = 2L_1 - L_2}$$

Hint **:**
(i) Apply Hooke's Law of elasticity.
(ii) Compare both the cases of loading and obtain relation.

Shortcut **:**
As loading is done on same wire
Thus, A, Y, L remains constant
Thus, Hooke's Law yield F α Δl

$$\Rightarrow \frac{F_1}{F_2} = \frac{\Delta l_1}{\Delta l_2}$$

$$\frac{1 \times g}{2 \times g} = \frac{L_1 - L}{L_2 - L}$$

$$\frac{1}{2} = \frac{L_1 - L}{L_2 - L}$$

$$L_2 - L = 2L_1 - 2L$$

Thus, $\boxed{L = 2L_1 - L_2}$

7. Option (B) is correct.

Explanation: In photoelectric effect ejection of electron from metal surface takes place only when :
Energy of incident photons is greater or equal to the Work Function of the Metallic Surface.
Where, work function is the minimum energy required to free an electron from metal surface.
∴ When energy of photon is less than work function, no electron will become free and photoelectric effect does not take place.
Thus, Assertion (A) is correct.
By equation of photoelectric effect

$h\nu = \phi + K.E_{max}$

As per Reason (R), if $K.E = 0$

Then $h\nu = \phi$

Thus, Reason (R) is also correct
But Reason (R) is not explaining Assertion (A) because it does not consider or explain why photoelectric effect does not take place if $h\nu < \phi$

Hint **: (i)** Use equation of photoelectric effect and check the correctness of both statements.
(ii) If the combined statement makes sense then Reason (R) is correct explanation of Assertion (A) otherwise not.

Shortcut **:**
To free the electrons from metal surface, minimum energy required is equal to the work function of metal.
So, Assertion (A) is correct.

$$h\nu = \phi + K.E_{max}$$

if $h\nu = \phi$

$\Rightarrow K.E_{max} = 0$

Hence, Reason R is correct.
But, Reason (R) does not explains Assertion (A)
Thus, option B is correct.

8. **Option (D) is correct.**

Explanation: Given: Velocity as a function of position as $v = bx^{5/2}$ ...(1)

By differentiating the above equation with respect to time

$$a = \frac{dv}{dt} = \frac{d}{dt}(bx^{5/2}) = \frac{5}{2}bx^{3/2}$$

$$\Rightarrow a = \frac{5}{2}bx^{3/2} \times v \qquad \left[\text{since, } v = \frac{dx}{dt}\right]$$

$$\Rightarrow a = \frac{5}{2}bx^{3/2} \times b \times x^{5/2} \Rightarrow a = \frac{5}{2}bx^4$$

Mass of body = 500 gm = 0.5 kg

Force acting on body = $m \times a$

(by Newton's II law of motion)

$$F = 0.5 \times \frac{5}{2}bx^2x^4 = \frac{5}{4}b^2x^4$$

By definition of work done $\Rightarrow$ W = $\int_{x_1}^{x_2} Fdx$

$$\Rightarrow W = \int_{x=0}^{x=4} \frac{5}{4}b^2x^4dx$$

$$\Rightarrow = \frac{5b^2}{4}\int_{x=0}^{x=4} x^4dx$$

$$\Rightarrow W = \frac{5}{4}\times\left(\frac{1}{4}\right)^2\left[\frac{x^5}{5}\right]_{x=0}^{x=4}$$

$$= \frac{\cancel{5}}{4}\times\frac{1}{16}\times\frac{4^5-0}{\cancel{5}} = \frac{1024}{64}$$

$$\Rightarrow W = 16 \text{ J}$$

***Hint* :**

(i) By using definition of work, $W = \int_{x_1}^{x_2} F\,dx$.

(ii) By differentiating v, obtain acceleration (a) hence find force.

(iii) Integrate with proper limits and obtain work done.

***Shortcut* :** As, velocity is given as a function of position Obtain velocity at x = 0 m and x = 4 m

Apply work-energy theorem

Net work done = Change in kinetic energy

$$W = \frac{1}{2}m[(v_{(4)})^2 - (v_{(0)})^2] \qquad \left[\because v(4) = \frac{1}{4}\times(4)^{5/2} = \frac{1}{4}\times(2)^5 = 8 \text{ m/s}\right.$$

$$= \frac{1}{2}\times\frac{1}{2}[(8)^2-(0)^2] \qquad \left.\because v(0) = \frac{1}{4}\times(0)^{5/2} = 0 \text{ m/s}\right]$$

$$= \frac{1}{4}\times 64 = 16 \text{ J}$$

9. **Option (C) is correct.**

Explanation: In a cyclotron, charge is rotated along circular path by magnetic field and it's kinetic energy is increased by electric field.

Radius of charged particle on circular path is given by

$$R = \frac{mv}{Bq}$$

Let the initial speed of charged particle be v

Due to application of electric field, it's kinetic energy increases to 4 times. Thus, new speed of charged particle

$$4K = \frac{1}{2}m(v)^2$$

$$\frac{v^2}{(v')^2} = \frac{1}{4} \Rightarrow \frac{v}{v'} = \frac{1}{2} \Rightarrow v = 2v$$

Thus, speed of charged particle becomes doubled.

Let initial radius of circular path be R_1 and final radius of circular path be R_2. Then,

$$R_1 = \frac{mv}{Bq} \qquad ...(1)$$

$$R_2 = \frac{m\times 2v}{Bq} \qquad ...(2)$$

from (1) and (2)

$$\boxed{R_2 = 2R_1}$$

Thus, Radius gets doubled.

***Hint* :**

(i) As radius of charged particle in uniform magnetic field, $R = \frac{mv}{Bq}$

(ii) Mass, charge of charged particle and magnetic field remains constant.

(iii) Thus, $R \propto v \Rightarrow \frac{R_1}{R_2} = \frac{v_1}{v_2}$

(iv) As, kinetic energy becomes 4 times of initial, then speed gets doubled.

Shortcut **:**

As, $R = \frac{mv}{Bq} \Rightarrow R = \frac{p}{Bq} - \sqrt{\frac{2mK}{Bq}}$

where $p \Rightarrow$ momentum of particle
$K \rightarrow$ kinetic energy of particle

Thus, $\frac{R_1}{R_2} = \sqrt{\frac{K_1}{K_2}}$

$$\frac{R_1}{R_2} = \sqrt{\frac{K}{4K}} = \frac{1}{2}$$

$$\boxed{R_2 = 2R_1}$$

10. Option (C) is correct.

Explanation: At resonance $X_L = X_C$ and impedance of circuit is minimum and is equal to R and current is maximum.

At resonance, $\omega_0 = \frac{1}{\sqrt{LC}}$

$\Rightarrow$ If potential drop across capacitor is more than inductor, then current leads voltage and circuit becomes capacitive.

$\Rightarrow$ If potential drop across inductor is more than capacitor, then voltage leads current and circuit becomes inductive

Thus, $i_0X_L > i_0X_C$	$i_0X_C > i_0X_L$
Circuit is inductive	Circuit is capacitive
$X_L > X_C$	$X_C > X_L$
$\omega L > \frac{1}{\omega C}$	$\frac{1}{\omega C} > \omega L$
$\omega^2 > \frac{}{\sqrt{LC}}$	$\omega < \frac{1}{\sqrt{LC}}$
$\omega > \sqrt{\frac{1}{\sqrt{LC}}}$	$\omega < \sqrt{\frac{1}{\sqrt{LC}}}$
$\omega > \omega_R$	$\omega < \omega_R$

Hint **:**

(i) At resonance, $\omega = \omega_R = \sqrt{\frac{1}{LC}}$, and $X_L = X_C$. Thus circuit is purely resistive and impedance is minimum.

(ii) $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$

On increasing $\omega\uparrow$, $X_L > X_C$ circuit is inductive.
On decreasing $\omega\downarrow$, $X_C > X_L$ circuit is capacitive.

Shortcut **:**

$X_L \propto \omega$ and $X_C \propto \frac{1}{\omega}$

On increasing ω, X_L increases and circuit becomes inductive.

On decreasing ω, X_L decreases and X_C increases circuit becomes capacitive.

11. Option (C) is correct.

Explanation: As water ejected by jet strikes the block, it imparts momentum to the block

$\Rightarrow$ Change in momentum of water jet in 1 s = Momentum gained by block in 1 s.

$\Rightarrow$ Change in momentum of jet in second = Mass of jet ejected in 1 second $\times$ change in velocity

$$= 1\,\frac{kg}{s} \times (10 - 0)$$
$$= 10 \text{ kg m/s}$$

$\Rightarrow$ Momentum gained by block = 10 kg m/s

By Newton's II law of motion $\Rightarrow$ F = Rate of change of momentum

$$= \frac{\Delta p}{\Delta t}$$

$$F = \frac{10\ kg\ m/s}{1s} = 10 \text{ kg } \frac{m}{s^2} = 10\ N$$

Thus, 10 N force acts on the block

By $F = ma$

$$a = \frac{F}{m} = \frac{10}{2} = 5\text{m/s}^2$$

Hint **:**

(i) Apply impulse-momentum equation on the block.

Shortcut **:**

As water is continuously ejecting out of jet.

Thus, it's a variable mass system

For variable mass system

$$F = v\frac{dm}{dt}$$

$$F = 10\,\frac{\text{m}}{\text{s}} \times \frac{1kg}{\text{s}}$$

$$F = 10 \text{ kg } \frac{\text{m}}{\text{s}^2} = 10 \text{ N}$$

By $F = ma \Rightarrow a = \frac{F}{m} = \frac{10}{2} = 5\text{m/s}^2$

12. Option (C) is correct.

Explanation: $\left[P+\frac{a}{V^2}\right]\left[V-b\right]=RT$

By principle of Dimensional Homogeneity, physical quantities with same dimensions only can be added or subtracted.

Thus, Dimensions of P and $\frac{a}{V^2}$ and dimensions of V and b must be same

$$\therefore \quad [P] = \left[\frac{a}{V^2}\right] \Rightarrow [a] = [PV^2] \quad ...(1)$$

$$[V] = [b]$$

$$\text{Thus,} \quad \text{Ratio of } \frac{a}{b} = \frac{[PV^2]}{[V]} = [PV]$$

Hint :

(i) Apply principle of Dimensional Homogeneity and obtain dimensions of a and b.

(ii) Calculate dimensions of $\frac{a}{b}$.

Shortcut :

As physical quantities of only same dimensions can be added or subtracted

$$[P] = \left[\frac{a}{V^2}\right] \Rightarrow [a] = [PV^2] \quad ...(1)$$

$$[V] = [b] \quad ...(2)$$

On dividing (1) by (2), we get:

$$\left[\frac{a}{b}\right] = \frac{\left[PV^2\right]}{V} = [PV]$$

13. Option (C) is correct.

Explanation: Given $|\vec{A}+\vec{B}|=2|\vec{A}-\vec{B}|$ ...(1)

Let angle between $\vec{A}$ and $\vec{B}$ be θ.

$$|A| = |B|$$

Magnitude of resultant of two vectors is given by

$$|\vec{R}| = \sqrt{A^2+B^2+2AB\cos\theta}$$

$$|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$$

$$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2+2AB\cos(\pi-\theta)}$$

$$= \sqrt{A^2+B^2-2AB\cos\theta}$$

Substituting values of $|\vec{A}+\vec{B}|$ and $|\vec{A} \quad \vec{B}|$ in equation (1), we get :

$$\sqrt{A^2+B^2+2AB\cos\theta} = 2\sqrt{A^2+B^2-2AB\cos\theta}$$

On squaring both the sides

$$(A^2+B^2+2AB\cos\theta) = 4(A^2+B^2-2AB\cos\theta)$$

$$\text{As} \quad |A| = |8| \Rightarrow A = B$$

$$2+2\cos\theta = 4\,(2-2\cos\theta)$$

$$6 = 10\cos\theta$$

$$\cos\theta = \frac{6}{10}$$

$$\therefore \quad \cos\theta = \frac{3}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{5}\right)$$

Hint :

(i) Apply the formula of resultant of addition of two vectors and subtraction of two vectors.

(ii) As $|\vec{A}+\vec{B}| = 2\,|\vec{A}-\vec{B}|$

(iii) Put $A = B$ and solve for θ

Shortcut : $|\vec{A}+\vec{B}| = 2\,|\vec{A}-\vec{B}|$

Squaring both the sides

$$|\vec{A}+\vec{B}|^2 = 4\,|\vec{A}-\vec{B}|^2$$

$$\text{Using identity } \vec{A}.\vec{A} = |\vec{A}|^2$$

$$(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = 4\,(\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B})\cdot$$

$$|A|^2+|B|^2+2\vec{A}\cdot\vec{B} = 4\,|A|^2+4|B|^2 - 8\vec{A}\cdot\vec{B}$$

$$10\,\vec{A}\cdot\vec{B} = 3\,[|A|^2+|B|^2]$$

$$10\,|A|\,|B|\cos\theta = 3\,(|A|^2\,|B|^2)$$

$$\text{As } |A| = |B|$$

$$10\cos\theta = 6$$

$$\cos\theta = \frac{6}{10} \Rightarrow \cos\theta = \frac{3}{5}$$

$$\theta = \cos^{-1}\left(\frac{3}{5}\right)$$

14. Option (A) is correct.

Explanation: Escape velocity of a body on a planet is given by :

$$V_e = \sqrt{2gR} \text{ or } V \sqrt{\frac{GM}{}}$$

where $g \Rightarrow$ acceleration due to gravity at the surface

$m \Rightarrow$ mass of planet and R = Radius of planet

Assuming plant to be solid sphere uniform, density ρ

$$\text{Mass} = \text{Density} \times \text{Volume}$$

$$M = \rho \times \frac{4}{3}\pi R^3$$

The expression for escape velocity can be written as

$$V_e = \sqrt{\frac{2G\rho \times \frac{4}{3}\pi R^3}{R}} = \sqrt{\frac{8\pi}{3}G\rho R^2}$$

(V_e) for planet A = 12 km/s.

Let it's Radius R_A and density ρ

For planet $B \Rightarrow (V_e)_B \Rightarrow R_B = \frac{R_A}{2}, \rho_B = 4\rho_A$

$$\frac{(V_e)_A}{(V_e)_B} = \frac{\sqrt{\frac{8\pi G\rho_A R_A{}^2}{3}}}{\sqrt{\frac{8\pi G\rho_B R_B{}^2}{3}}}$$

$$= \sqrt{\frac{\rho_A}{\rho_B}\left(\frac{R_A}{R_B}\right)^2} = \sqrt{\frac{1}{4} \times 4} = 1$$

$$\therefore \quad (V_e)_A = (V_e)_B = 12 \text{ km/s}$$

Hint **:**

(i) Use the formula of escape velocity

$$V_e = \sqrt{\frac{2GM}{R}}$$

and convert mass in terms of density and volume.

(ii) Obtain relation of escape speed in terms of radius and density.

(iii) Compare escape velocity of both planets.

Shortcut **:** $V_e \propto \sqrt{\frac{m}{R}}$

for a solid sphere

$$\Rightarrow \quad m = \frac{4}{3}\pi R^3 \times \delta$$

$$\therefore \quad m \propto \rho \text{ and } m \propto R^3$$

Thus, $V_e \propto \sqrt{\frac{\rho R^3}{R}} \propto \sqrt{\rho R^2}$

$$\Rightarrow \quad V_e \propto R\sqrt{\rho}$$

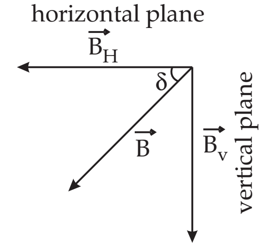
$$\Rightarrow \quad \frac{V_A}{V_B} = \frac{R_A}{R_B}\sqrt{\frac{\rho_A}{\rho_B}} \Rightarrow \sqrt{\frac{\rho_A}{4\rho_B}}$$

$$\Rightarrow \quad \frac{V_A}{V_B} = 2 \times \sqrt{\frac{1}{4}} = 2 \times \frac{1}{2} = 1$$

$$\therefore \quad V_A = V_B = 12 \text{ km/s}$$

15. Option (A) is correct.

Explanation: Angle of dip at a place is defined as angle between earth's magnetic field and horizontal plane at that location.



$\delta \rightarrow$ Angle of Dip

On resolving $\vec{B}$ into two perpendicular components

$$B_H = B\cos\delta \quad \text{...(1)}$$

$$B_V = B\sin\delta \quad \text{...(2)}$$

Given horizontal component of earth's magnetic field at a location is 0.5 G

Using equation (1)

$$0.5 = B\cos 30°$$

$$0.5 = B \times \frac{\sqrt{3}}{2}$$

$$B = \frac{0.5 \times 2}{\sqrt{3}} = \frac{1}{\sqrt{3}} G$$

Hint **:**

Use the relation $B_H = B\cos\delta$

Shortcut **:** As, $B_H = B_H \cos\theta$

Given, $B_H = 0.5\,G$

$\delta = 30°$

Hence, $0.5\,G = B \times \cos 30°$

$$B = \frac{0.5}{\cos\delta} = \frac{0.5}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \text{G}$$

16. Option (B) is correct.

Explanation: $y = 10\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$

$$y = 10\sin\left(2\pi nt - \frac{2\pi}{\lambda}x\right)$$

On comparing it with standard equation of wave

$$\Rightarrow \omega\ 2\pi n \text{ and } k = \frac{2\pi}{\lambda}$$

Given : Maximum particle velocity = 4 × wave velocity

In wave propagation, medium particle oscillates and maximum velocity of oscillating particle is equal to $A\omega$

Where, $A \Rightarrow$ Amplitude

$\omega \Rightarrow$ Angular frequency

$\Rightarrow$ Wave velocity is given by $V = \dfrac{\omega}{k}$ where, k is angular wave number and $k = \dfrac{2\pi}{\lambda}$

$$V_{\text{wave}} = \frac{2\cancel{\pi}n}{2\cancel{\pi}\,\lambda} = n\lambda$$

$$V_{\text{particle}} = 4\,V_{\text{wave}}$$

$$A\omega = 4n\lambda$$

$$A \times 2\pi n = 4n\lambda$$

$$A \times 2\,\pi n = 4n\lambda$$

$$\lambda = \frac{2\pi A}{4} = \frac{\pi A}{2}$$

As, $\quad A = 10 \Rightarrow \lambda = \dfrac{\pi \times 10}{2} = 5\pi$

Hint **:**

(i) Compare the given equation of wave with standard equation of wave $y = \sin(\omega t - kx)$

(ii) Obtion ω, k

(iii) Find V_{wave} and substitute in the given situation i.e., $V_{\text{particle}} = 4\,V_{\text{wave}}$

Shortcut **:**

$$y = 10\sin\left(2\pi nt - \frac{2\pi x}{\lambda}\right)$$

Standard equation of wave $y = A\sin(\omega t - kx)$

$\Rightarrow \omega = 2\pi,\ nk = \dfrac{2\pi x}{\lambda}$

Maximum velocity of oscillating particle = $A\omega$

$$A\omega = 4\frac{\omega}{k}$$

$$A = \frac{4}{\frac{2\pi}{\lambda}} \Rightarrow \lambda = \frac{2\pi A}{4}$$

$$= \frac{\pi A}{2} = \frac{\pi \times 10}{2} = 5\pi$$

Thus $\quad \lambda = 5\pi$

17. Option (A) is correct.

Explanation: Potential energy stored in air capacitor of capacitance C, charged to potential difference V is given by:

$$U = \frac{1}{2}CV^2$$

If the above capacitor is filled completely with a dielectric of dielectric constant K, keeping the potential difference same, then potential energy of capacitor becomes

$$U' = \frac{1}{2}KCV^2 = KU$$

Given:

Initially K of dielectric = 10

$U_i \Rightarrow$ Initial potential energy = $\dfrac{1}{2} \times 10 \times CV^2$

$U_f \Rightarrow$ Final potential energy = $\dfrac{1}{2} \times 15 \times CV^2$

percentage increase in energy of capacitor

$$= \frac{U_f - U_i}{U_i} \times 100$$

$\Rightarrow$ percentage increase

$$= \frac{\frac{1}{2}CV^2\,[15-10]}{\frac{1}{2}CV^2 \times 10} \times 10$$

$$= \frac{5}{10} \times 100 = 50\%$$

Hint:

(i) Use the expression of potential energy stored in a capacitor filled with dielectric i.e.,

$$U = \frac{1}{2}KCV^2$$

(ii) Compare energy stored in both cases.

Shortcut:

As potential difference across capacitor remains same in both cases.

$$U = \frac{1}{2}KCV^2$$

$\Rightarrow \quad U \propto K$

$\Rightarrow \quad \dfrac{U_2}{U_1} = \dfrac{K_2}{K_1}$

$\Rightarrow \quad \dfrac{U_2 - U_1}{U_1} = \dfrac{K_2 - K_1}{K_1}$

$\Rightarrow \quad \dfrac{U_2 - U_1}{U_1} \times 100 = \dfrac{K_2 - K_1}{K_1} \times 100$

$\Rightarrow$ percentage increase in $U = \dfrac{15-10}{10} \times 100$

$= 50\%$

18. Option (D) is correct.

***Explanation:* Given:**

u = Initial velocity of charged particle = 200 m/s
m = mass of charged particle = 100 mg
$= 100 \times 10^{-6}$ kg $= 10^{-4}$ kg
$q \Rightarrow$ charge on particle = 40 μC = $40 \times 10^{-6}\ C = 4 \times 10^{-5}\ C$
$|E|$ = Electric field intensity = $1 \times 10^5\ N/C$

As particle is projected opposite to the direction of electric field, the electrostatic force will act on the particle opposite to velocity. Thus, its motion will be retarded.

As electric field is constant, $F = qE$ will be constants

Hence, $a = \frac{F}{m} = \frac{qE}{m}$ will also be constant.

Thus, equations of motion can be used

$$v^2 = u^2 + 2as \quad \ldots(1)$$

$$(0)^2 = (200)^2 + 2 \times \left(\frac{-qE}{m} \times S\right)$$

$$\frac{2 \times 4 \times 10^{-5} \times 1 \times 10^5}{10^{-4}} \times S = 4 \times 10^4$$

$$S = \frac{4 \times 10^4 \times 10^{-4}}{8} = \frac{1}{2}$$

$$m = 0.5 \text{ m}$$

Thus, $S = 0.5$ m

Hint:

(i) As electric field is uniform, if will apply a constant force, F = qE on charged particle.
Then, use $a = \frac{qE}{m}$

(ii) Use III equation of motion i.e., $v^2 = u^2 + 2as$ to calculate the required stopping distance.

Shortcut : By Work-Energy Theorem:

$\Rightarrow$ Work done = change in Kinetic energy
As force acting on charged particle is constant,

$$F = qE$$

Let the stopping distance by S.

$$K_{\text{inital}} = \frac{1}{2} \times m\,(200)^2$$

$$K_{\text{final}} = 0$$

$$\Rightarrow \quad W = K.E_f - K.E_i$$

$$qE \times S = +\,K.E_i$$

$$S = \frac{10^{-4} \times (200)^2}{2 \times 4 \times 10^{-5} \times 1 \times 10^5}$$

$$\Rightarrow \quad S = 0.5 \text{ m}$$

19. Option (D) is correct.

Explanation: As, $\beta = \frac{\lambda D}{d}$

Case 1: $0.5 \text{ mm} = \frac{5000\text{Å} \times D}{d_1} \quad \ldots(1)$

Case 2: $\beta_2 = \frac{6000\text{Å} \times D}{2d_1} \quad \ldots(2)$

Dividing Eq. (1) by (2), we get

$$\frac{0.5}{\beta_2} = \frac{5000}{6000} \times \frac{2d_1}{d_1}$$

$$\beta_2 = \frac{3}{10} = 0.3 \text{ mm}$$

Hint:

(i) Use the expression of fringe width in double slit $\beta \propto \frac{\lambda D}{d}$

(ii) Compare β for both the cases

Shortcut: Fringe width in double slit experiment = $\frac{\lambda D}{d}$

As D is kept constant

$$\beta \propto \frac{\lambda}{d}$$

Thus, $$\frac{\beta_1}{\beta_2} = \left(\frac{\lambda_1}{\lambda_2}\right)\left(\frac{d_2}{d_1}\right)$$

$$\Rightarrow \quad \frac{0.5 \text{ mm}}{\beta_2} = \frac{5000}{6000} \times \frac{2}{1}$$

$$\Rightarrow \quad \beta_2 = \frac{3}{10} \text{ mm} = 0.3 \text{ mm}$$

20. Option (B) is correct.

***Explanation:* Given:**

Wavelength of signal = 1000 mm = $10^3 \times 10^{-9}$
$= 10^{-6}$ m

c = Speed of light = 3×10^8 m/s

As $c = n\lambda$

$$n = \frac{c}{\lambda} = \frac{3 \times 10^8}{10^{-6}} = 3 \times 10^{14} \text{ Hz}$$

Thus, frequency of optical signal is 3×10^{14} Hz

Given only 2% of optical source frequency is available for communication system.

$= 0.75 \times 10^9 = 75 \times 10^7$

$\therefore$ Available frequency $= \dfrac{2}{100} \times 3 \times 10^{14}$

$= 6 \times 10^{12}$ Hz

Bandwidth of communication system

$= 6 \times 10^{12}$ Hz

Bandwidth of each channel to be accommodated

$= 8$ kHz $= 8 \times 10^3$ Hz

Thus, number system of channel

$$= \frac{\text{Bandwidth of system}}{\text{Bandwidth of each channel}}$$

$$n = \frac{6\times10^{12}}{8\times10^3}$$

$$= 0.75 \times 10^9 = 75 \times 10^7$$

Hint:

(i) Calculate the frequency of communicating system by $c = n\lambda$

(ii) By given percentage, calculate the available bandwidth of frequency.

(iii) Use expression n

$$= \frac{\text{Available Bandwidth}}{\text{Bandwidth of one channel}}$$

Shortcut:

Frequency of signal having wavelength 1000 nm is

$$\frac{c}{\lambda} = \frac{3\times10^8}{1000\times10^{-9}} = 3 \times 10^{14} \text{ Hz}$$

$\Rightarrow$ Frequency available for communication

$$= \frac{2}{100} \times 3 \times 10^4 = 6 \times 10^{12} \text{ Hz}$$

Bandwidth for one channel = 8000 Hz

$\therefore$ Number of channel $= \dfrac{6\times10^{12}}{8\times10^3} = 75 \times 10^7$

21. Correct answer is [15].

Explanation: Let there be two coils A and B.

Let the potential difference across source be V

Using $\quad H = \dfrac{V^2}{R}t$

For Coil (1) $\quad H = \dfrac{V^2}{R_1} \times 60 \quad$ …(1),

For Coil (2) $\quad H = \dfrac{V^2}{R_2} \times 20 \quad$ …(2)

When coils (1) and (2) connected in parallel combination across voltage source.

Total Heat generated $= H_1 + H_2$

Let the total time be 't' when two coils together in parallel are operated.

As $\quad H_{\text{total}} = H$

$$\Rightarrow \quad H = \frac{V^2}{R_1}t + \frac{V^2}{R_2}t$$

$$H = \frac{H}{60}t + \frac{H}{20}t$$

From equation (1) and (2)

$$1 = \left(\frac{1}{60} + \frac{1}{20}\right)t$$

$$\Rightarrow \quad t = \frac{1200}{80} = 15 \text{ minutes}$$

Hint:

(i) Use expression $H = \dfrac{V^2}{R}t$ to calculate heat generated, as potential drop across each coil is same.

(ii) As coils are combined in parallel.

Apply $H_{\text{total}} = H_{\text{coil (1)}} + H_{\text{coil (2)}}$

Shortcut: As heat generated in both coil is same

i.e., $\quad \dfrac{V^2}{R_1} \times 60 = \dfrac{V^2}{R_2} \times 20$

$$\frac{3}{1} = \frac{R_1}{R_2}$$

Assume $R_1 = 3\Omega$, $R_2 = 1\Omega$

Then, $\quad \dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{3} + \dfrac{1}{1} = \dfrac{4}{3}$

And, $\quad \dfrac{V^2}{R_{eq}} \times t = \dfrac{V^2}{R_1} \times 60$

$$t = \frac{R_{eq}}{R_1} \times 60 = \frac{3}{4\times3} \times 60$$

$= 15$ minutes

22. Correct answer is [43].

Explanation: As $I = \left(\dfrac{1}{2}\varepsilon_0 E_0^2\right)$

On substituting value of I, ε_0 and c, we get

$$0.22 = \frac{1}{2}(8.85 \times 10^{-12})\, E_0^2 \times 3 \times 10^8$$

$$E_0 = \sqrt{\frac{2\times0.22}{8.85\times10^{-12}\times3\times10^8}}$$

$$= 12.9 \text{ N/C}$$

As $c = \dfrac{E_0}{B_0}$

$$\Rightarrow \quad B_0 = \frac{12.9}{3\times10^8} = 4.3\times10^{-8} = 43\times10^{-9}\ T$$

Hint:

(i) Apply relation of intensity i.e., $\frac{1}{2}\varepsilon_0 E_0^2 \times c$ and calculate E_0.

(ii) As, speed of light $c = \dfrac{E_0}{B_0}$, B_0 can be calculated.

Shortcut: $I = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)\times c = \frac{1}{2}\frac{B_0^2}{\mu_0}c$

$$0.22 = \frac{1}{2}\frac{B_0^2\times3\times10^8}{4\pi\times10^{-7}}$$

$$B_0 = \sqrt{\frac{0.22\times2\times4\times3.14\times10^{-7}}{3\times10^8}}$$

$$= 4.3\times10^{-8} = 43\times10^{-9}\ T$$

23. Correct answer is [21].

Explanation: Since, $\dfrac{Q}{t} = \dfrac{KA\Delta T}{d}$

Here, all alphabets are is their usual meanings

where $\dfrac{Q}{t}$ = Rate of heat transfer

$$\Rightarrow \quad \frac{Q_1}{t} = \frac{Q_2}{t}$$

$$\Rightarrow \quad \frac{100-T_i}{4/KA} = \frac{T_i-0}{2.5/2\ KA}$$

$$\Rightarrow \quad \frac{100-T_i}{4} = \frac{2T_i}{2.5}$$

$$\Rightarrow \quad 250-2.5\ T_i = 8\ T_i$$

$$T_i = \frac{250}{10.5}$$

As the two plates are in series, same thermal current will flow in composite rod as well

DT for composite plate = 100°C

$$\Rightarrow \quad \frac{\dfrac{100-0}{6.5}}{K_{eq}A} = \frac{\dfrac{250}{10.5}-0}{\dfrac{2.5}{2\ KA}}$$

$$\Rightarrow \quad \frac{K_{eq}\times100}{6.5} = \frac{250\times2K}{10.5\times2.5}$$

$$K_{eq} = \frac{130}{105}K \quad \frac{26}{21}K = \left(\frac{1+5}{21}\right)K$$

on comparing with $K_{eq} = \left(1+\dfrac{5}{21}\right)K$, we get

$$\alpha = 21$$

Hint:

(i) The two plates of same cross-section area are connected in series.

(ii) As thermal conduction obeys Ohm's Law, the two plates act as thermal resistors which are connected in series.

(iii) Use $R_{eq} = R_1 + R_2$

Hence, $\dfrac{L_{eq}}{K_{eq}A} = \dfrac{L_1}{K_1 A} + \dfrac{L_2}{K_2 A}$

$$\Rightarrow \quad K_{eq} = \frac{(K_1K_1)\ L_{eq}}{L_1K_1 + L_2K_1}$$

Shortcut: As two plates are connected in series Thus, same thermal current (heat transfer $\mathring{Q}$)

$\Rightarrow$ Thermal Resistance also follow same combination laws as that of electrical resistors. In above diagram, compound plate is made by joining two plates in series

$$\therefore \quad R_{th} = R_{th_1} + R_{th_2}$$

$$\frac{L_1+L_2}{K_{eq}A} = \frac{L_1}{K_1A} + \frac{L_2}{K_2A}$$

Thus, $\dfrac{1}{K_{eq}} = \dfrac{L_1K_2 + L_2K_1}{K_1K_2\ (L_1+L_2)}$

$$\Rightarrow \quad \frac{1}{K_{eq}} = \frac{4\times2K+2.5\times K}{2K^2\times6.5} = \frac{10.5K}{13K^2}$$

$$\Rightarrow \quad K_{eq} = \frac{13K}{10.5} = \frac{130}{105}K$$

$$= \frac{26}{21}K = \left(1+\frac{5}{21}\right)K$$

24. Correct answer is [700].

Explanation: Let the equation of particle performing SHM be:

$$x = A\sin(\omega t) \qquad \ldots\text{(i)}$$

On differentiating equation (1) we get,

$$v = \frac{dx}{dt} = A\omega\cos\omega t$$

$$\Rightarrow \quad v = a\omega\sqrt{1-\sin^2 \omega t}$$

$$\Rightarrow \quad v = A\omega\sqrt{1-\left(\frac{x}{A}\right)^2} \text{ [From eq (1)]}$$

$$v = \omega\sqrt{A^2 - x^2}$$

Initially, the amplitude of SHM is 10 cm.

When the particle is at a location of 5 cm i.e., $\frac{A}{2}$, it's speed is increased by air jet to 3V.

$\Rightarrow$ This will increase the total energy of SHM and and hence amplitude will also increase.

$\Rightarrow$ Angular frequency i.e., $\omega = \sqrt{\frac{\text{Elastic Factor}}{\text{Inertial factor}}}$

remains same because neither mass has changed nor any force.

$$v = \omega\sqrt{A^2 - \left(\frac{A}{2}\right)^2} \qquad \left[\text{When } x = \frac{A}{2}\right] \ldots(1)$$

$$3v = \left[\omega\sqrt{(A')^2 - \left(\frac{A}{2}\right)^2}\right]\left[\text{at } x = \frac{A}{2}\right] \qquad \ldots(2)$$

Divide (1) by (2)

$$\frac{1}{3} = \sqrt{\frac{A^2 - \frac{A^2}{4}}{A'^2 - \frac{A^2}{4}}}$$

On Sauaring both sides, we get;

$$\frac{1}{9} = \frac{\frac{3A^2}{4}}{A'^2 - \frac{A^2}{4}}$$

$$\Rightarrow \quad A'^2 - \frac{A^2}{4} = \frac{27A^2}{4}$$

$$\Rightarrow \quad A'^2 = \left(\frac{27}{4} + \frac{1}{4}\right)\frac{A^2}{4} = 7A^2$$

$$\Rightarrow \quad A' = \sqrt{7}A$$

On comparison with $\sqrt{x}$

$$x = 700 \text{ [As, } A = 10 \text{ cm]}$$

Hint:

(i) Use the relation $v = \omega\sqrt{A^2 - x^2}$

(ii) Write velocity in both cases in terms of amplitude and solve them to find new amplitude.

Shortcut: Kinetic energy of an oscillating particle is gives as $\Rightarrow K = \frac{1}{2}m\omega^2(A^2 - x^2)$

$\Rightarrow$ As velocity is tripled, kinetic energy becomes g times

$$\frac{K}{9K} = \frac{A^2 - \frac{A^2}{4}}{A'^2 - \frac{A^2}{4}}$$

$$\Rightarrow \quad A'^2 - \frac{A^2}{4} = 9A^2 - \frac{9A^2}{4}$$

$$\Rightarrow \quad A'^2 = \frac{28A^2}{4}$$

$$= 7A^2$$

$$\Rightarrow \quad A' = \sqrt{7}A$$

$$\Rightarrow \quad A' = \sqrt{7} \times 10 \text{ cm}$$

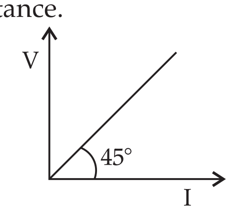
$$\Rightarrow \quad A' = \sqrt{700} \text{ cm}$$

By comparing it with given value, we get;

$$\therefore \quad x = 700$$

25. Correct answer is [144].

Explanation: We know that slope of *V-I* graph gives Resistance.



Thus, Slope $= R = \tan\theta = \tan 45° = 1$

Thus, $R = 1\Omega$

By using $R = \frac{\rho l}{A}$

$$\Rightarrow \quad 1 = \frac{x \times 10^{-5} \times 31.4 \times 10^{-2}}{\frac{\pi \times (2.4 \times 10^{-2})^2}{4}}$$

$$\Rightarrow \quad x = \frac{2.4 \times 2.4 \times 10^{-4}}{4 \times 10^{-6}} = \frac{2.4 \times 2.4 \times 10^2}{4}$$

$$x = \frac{24 \times 24}{4}$$

$$= 144$$

Hint:

(i) By using slope of *V-I* graph, find resistance.

(ii) Use Relation $R = \frac{\rho l}{A}$ to evaluate x

Shortcut:

By slope of V-I graph, $R = 1\Omega$ and $R = \frac{\rho l}{A}$

$$\Rightarrow \quad x \times 10^{-5} = \frac{3.14\times(2.4\times10^{-2})^2}{31.4\times10^{-2}\times4}$$

$$\Rightarrow \quad x = \frac{2.4\times2.4}{4}\times10^2 \Rightarrow x = 144$$

26. Correct answer is [102].

Explanation: Assuming the engine to be Carnot engine

$$\therefore \quad \frac{Q_{\text{absorbed}}}{Q_{\text{lost}}} = \frac{T_{\text{source}}}{T_{\text{sink}}} \quad \ldots(1)$$

On substituting values in equation (1)

$$\frac{300 \text{ cal}}{225 \text{ cal}} = \frac{500K}{T_{\text{sink}}}$$

$$T_{\text{sink}} = \frac{225}{300} \times 500 = 375\ K$$

$$T_{\text{sink}} (\text{in } °\text{C}) = 375 - 273 = 102\ °\text{C}$$

Hint:

(i) Use the relation, $\frac{T_H}{T_L} = \frac{Q_{\text{absorbed}}}{Q_{\text{Lost}}}$

(ii) Substitute the required values in above expression.

Shortcut: As engine operates on cyclic process

$$Q_{\text{net}} = W_{\text{net}}$$

$$\therefore \quad W_{\text{net}} = +\,300 - 225 = 75 \text{ cal}$$

$\eta \Rightarrow$ efficiency of Carnot engine $= 1-\frac{T_L}{T_H}$

$$\eta \Rightarrow \quad \frac{W}{Q_{\text{absorbed}}} = 1-\frac{T_L}{T_H}$$

$$\frac{75}{300} = 1-\frac{T_L}{T_H}$$

$$\frac{T_L}{500} = \frac{3}{4}$$

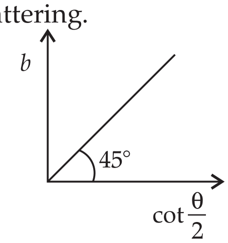
$$\Rightarrow \quad T_L = 375\ K$$

$$T_L = 375 - 273 = 102°\text{C}$$

27. Correct answer is [3].

Explanation: $\cot\frac{\theta}{2} \propto b$

i.e., more the impact parameter lesser is the angle of scattering.



Given: $\sqrt{d_1}$ = impact parameter in 1st case

$\theta_1 = 60°$

$\sqrt{d_2}$ = impact parameter in 2nd case

$\theta_2 = 90°$

$$\text{Hence,} \quad \frac{\cot\frac{\theta_1}{2}}{\cot\frac{\theta_2}{2}} = \sqrt{\frac{d_1}{d_2}}$$

On squaring both sides, we get;

$$\Rightarrow \quad \frac{\cot^2\frac{\theta_1}{2}}{\cot^2\frac{\theta_2}{2}} = \frac{d_1}{d_2}$$

$$\Rightarrow \quad \frac{(\sqrt{3})^2}{1} = \frac{d_1}{d_2}$$

$$\Rightarrow \quad d_1 = 3d_2$$

Hint:

(i) As $\cot\frac{\theta}{2} \propto b$

$$\frac{\cot\frac{\theta_1}{2}}{\cot\frac{\theta_2}{2}} = \frac{b_1}{b_2}$$

(ii) Put data in avobe relation and solve.

Shortcut: $\sqrt{d} \propto \cot\frac{\theta}{2}$

$$d_1 = xd_2$$

$$\cot^2\left(\frac{60°}{2}\right) = x\cot^2\left(\frac{90°}{2}\right)$$

$$\cot^2 30° = x\cot^2 45°$$

$$3 = x \times 1$$

Thus, $x = 3$

28. Correct answer is [2].

Explanation: Power gain is defined as the ratio of output power to the input power.

$$\text{Power gain} = \frac{\text{Output power}}{\text{Input power}}$$

$$= \frac{I^2_{\text{output}} \times R_{\text{output}}}{I^2_{\text{input}} \times R_{\text{input}}}$$

$$\frac{\Delta i_c}{\Delta i_b} = \frac{I_{\text{output}}}{I_{\text{input}}} = \beta = \text{current gain}$$

in common emitter mode.

$$\beta = \frac{10^{-2}A}{10^{-4}A} = 100$$

$$\therefore \quad \text{Power gain} = (\beta)^2 \times \frac{R_{\text{out}}}{R_{\text{in}}}$$

$$= 10^4 \times \frac{2\times10^3}{1\times10^3} = 2 \times 10^4$$

Thus, value of $x = 2$

Hint:

(i) In Common-emitter mode:

$$\text{Current gain, } \beta = \frac{\Delta I_c}{\Delta I_b} = \frac{10\text{ m}A}{100\ \propto A}$$

$$= 10^2 = 100$$

(ii) $$P_{gain} = \beta^2 \times \frac{R_{out}}{R_{in}}$$

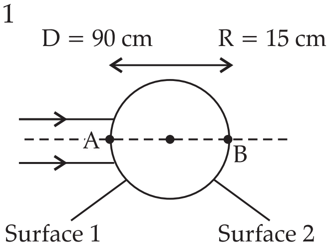
Shortcut:

$$\Delta I_b = 100\ \mu A = 10^{-4} A$$
$$\Delta I_c = 10\ mA = 10^{-2}\ A$$
$$\beta = \frac{\Delta I_c}{\Delta I_b} = 10^2$$
$$P_{gain} = \beta^2 = \frac{R_{out}}{R_{in}} = (10^2)^2 \times \frac{2}{1}$$
$$= 2 \times 10^4$$

Thus, $x = 2$

29. Correct answer is [225].

Explanation: Parallel Beam of light strikes at surface 1



Thus, 1[st] refraction takes place at surface 1 and, 2[nd] refraction takes place at surface 2

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Refraction at surface 1

All distances are measured from A

$$\Rightarrow \frac{1.5}{v} - \frac{1}{-\infty} = \frac{1.5-1}{+15}$$

[**Given:** $u = -\infty$, $v = ??$
$\mu_2 = 1.5$, $\mu_1 = 1$
$R = +15$ cm]

$$\frac{1.5}{v} = \frac{0.5}{15}$$

$$V = +45\text{ cm}$$

for refraction at surface 2

All distance will be measured from B

Image after refraction at surface 1 will play role of object for surface 2

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{1.5}{+15} = \frac{1-1.5}{-15}$$

$$\frac{1}{v} - \frac{1}{10} = \frac{0.5}{-15}$$

$$\frac{1}{v} = \frac{1}{30} + \frac{1}{10}$$

$$v = \frac{300}{40} = 7.5\text{ cm}$$

Thus, distance of final image from B is 7.5 cm

Thus, beam of light converges at point 15 + 7.5= 22.5 cm = 225 mm from centre of sphere.

Hint:

(i) Use the relation, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

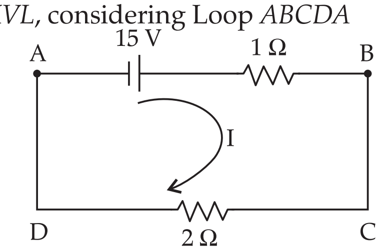
(ii) Apply the above equation two times first for surface 1 and then for surface 2. Obtain the final position image.

30. Correct answer is [10].

Explanation: By Kirchhoff's Loop Rule,

Potential difference while traversing a closed loop = 0

By *KVL*, considering Loop *ABCDA*



$$+ 15 - I \times 1 - 2 \times I = 0$$
$$15 = 3I$$
$$\Rightarrow \quad I = 5A$$

Now, $V_A + 15 - 1 \times 5 = V_B$

$$V_B - V_A = 15 - 5 = 10\ V$$

Hint:

(i) Apply Kirchhoff's Voltage Law in the given loop and find current.

(ii) Find potential difference across the branch AB.

Shortcut:

$V_B - V_A$ = Potential Difference across 2 Ω resistor

As 2 Ω and 1 Ω are connected in series.

$R_{aq} = 2 + 1 = 3\Omega$

The $$I_{circuit} = \frac{V}{R_{eq}} = \frac{15}{3} = 5A$$

Thus, by using Ohm's law, the potential difference across 2 Ω is $V_B - V_A = 2 \times 5 = 10\ V$

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